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Distance and Similarity Measures of Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Matrices and Their Applications to Data Classification in Supervised Learning

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Abstract: Intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices), proposed by Enginoğlu and Arslan in 2020, are worth utilizing in data classification in supervised learning due to coming into prominence with their ability to model decision-making problems. This study aims to define the concepts metrics, quasi-, semi-, and pseudo-metrics and similarities, quasi-, semi-, and pseudo-similarities over *ifpifs*-matrices; develop a new classifier by using them; and apply it to data classification. To this end, it develops a new classifier, i.e., Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Classifier (IFPIFSC), based on six pseudo-similarities proposed herein. Moreover, this study performs IFPIFSC's simulations using 20 datasets provided in the UCI Machine Learning Repository and obtains its performance results via five performance metrics, accuracy (Acc), precision (Pre), recall (Rec), macro F-score (MacF), and micro F-score (MicF). It also compares the aforementioned results with those of 10 well-known fuzzy-based classifiers and 5 non-fuzzy-based classifiers. As a result, the mean Acc, Pre, Rec, MacF, and MicF results of IFPIFSC, in comparison with fuzzy-based classifiers, are 94.45%, 88.21%, 86.11%, 87.98%, and 89.62%, the best scores, respectively, and with non-fuzzy-based classifiers, are 94.34%, 88.02%, 85.86%, 87.65%, and 89.44%, the best scores, respectively. Later, this study conducts the statistical evaluations of the performance results using a non-parametric test (Friedman) and a post hoc test (Nemenyi). The critical diagrams of the Nemenyi test manifest the performance differences between the average rankings of IFPIFSC and 10 of the 15 are greater than the critical distance (4.0798). Consequently, IFPIFSC is a convenient method for data classification. Finally, to present opportunities for further research, this study discusses the applications of *ifpifs*-matrices for machine learning and how to improve IFPIFSC.

Keywords: intuitionistic fuzzy sets; soft sets; *ifpifs*-matrices; distance measures; similarity measures; machine learning

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1. Introduction

Fuzzy sets [1,2] are a mathematical tool put forward by Zadeh to overcome the problems involving uncertainties in which classical sets are insufficient in modeling. Another tool offered to model problems involving uncertainties is soft sets [3–5]. Thus far, several hybrid versions of these two concepts have been defined, such as fuzzy soft sets [6] and fuzzy parameterized fuzzy soft sets (*fpfs*-sets) [7]. Recently, *fpfs*-sets have come to the fore due to their ability to model situations where both parameters and alternatives (objects) have fuzzy values. Afterward, fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) [8] have been defined to benefit from the modeling capabilities of *fpfs*-sets and avoid their disadvantages in decision-making problems containing a large amount of data.

Latterly, Memiş et al. [9] proposed a classifier, named Fuzzy Parameterized Fuzzy Soft Normalized Hamming Classifier (FPFSNHC), by defining normalized Hamming pseudo-similarity over *fpfs*-matrices and successfully applied it to classify some known datasets, such as "Breast Cancer Wisconsin (Diagnostic)", "Immunotherapy", "Pima Indian Diabetes", and "Statlog Heart". In addition, Memiş and Enginoğlu [10] have developed Fuzzy Parameterized Fuzzy Soft Chebyshev Classifier (FPFSCC) by defining Chebyshev pseudo-similarity over *fpfs*-matrices and successfully applied it to a classification problem containing medical datasets, such as "Cryotherapy", "Diabetic Retinopathy", "Hepatitis", and "Immunotherapy". Furthermore, Memiş et al. [11] have suggested a classifier using Euclidean pseudo-similarity over *fpfs*-matrices, namely Fuzzy Parameterized Fuzzy Soft Euclidean Classifier (FPFS-EC), and successfully applied it to a numerical data classification problem involving the datasets "Breast Tissue" and "Parkinson's Disease". Moreover, Memiş et al. [12,13] have propounded Fuzzy Parameterized Fuzzy Soft Aggregation Classifier (FPFS-AC) and Comparison Matrix-Based Fuzzy Parameterized Fuzzy Soft Classifier (FPFS-CMC) utilizing soft decision-making (SDM) methods. Thus, they have given a point of view of classifier constructions. In addition, Memiş et al. [14] have introduced a classifier named Fuzzy Parameterized Fuzzy Soft *k*-Nearest Neighbor (FPFS-*k*NN) and compared it with the *k*NN-based classifiers. The authors have used Pearson, Spearman, and Kendall correlation coefficients in the construction of FPFS-*k*NN, and these three constructions have been denoted by FPFS-*k*NN(P), FPFS-*k*NN(S), and FPFS-*k*NN(K), respectively.

Despite these successes of *fpfs*-matrices, they cannot model intuitionistic fuzzy uncertainties [15,16]. Therefore, intuitionistic fuzzy soft sets (*ifs*-sets) [17], intuitionistic fuzzy parameterized soft sets (*ifps*-sets) [18], and intuitionistic fuzzy parameterized fuzzy soft sets (*ifpfs*-sets) [19] have been studied. Later, the concept intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets (*ifpifs*-sets) [20], which can model situations where both parameters and objects with intuitionistic fuzzy values, has been defined and successfully applied to an SDM problem. Thereafter, intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices) [21] has been proposed and successfully applied to two SDM problems.

This paper focuses on developing a new classifier in data classification in supervised learning by operationalizing *ifpifs*-matrices and making theoretical contributions to them. The major contributions of the present study can be summarized as follows:

- ✓ Defining the concepts metrics, quasi-, semi-, and pseudo-metrics and similarities, quasi-, semi-, and pseudo-similarities over *ifpifs*-matrices.
- ✓ Proposing five pseudo-metrics and seven pseudo-similarities.
- ✓ Developing a new classifier, i.e., Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Classifier (IFPIFSC), with the best scores.
- ✓ Applying IFPIFSC to real-life classification problems successfully.

In the second part of this study, some basic definitions are required for the following sections are provided. Section 3 defines the metric, quasi-, semi-, and pseudo-metric over the *ifpifs*-matrices space and proposes five pseudo-metrics. In addition, it defines the concepts similarity, quasi-, semi-, and pseudo-similarity over the *ifpifs*-matrices space and suggests seven pseudo-similarities. Furthermore, this section clarifies some basic properties of the proposed five pseudo-metrics and seven pseudo-similarities. Section 4 proposes a classifier, i.e., IFPIFSC, based on multiple pseudo-similarities and presents the definitions used in the construction of IFPIFSC. Section 5 first provides the properties of 20 datasets in the UCI Machine Learning Repository (UCI-MLR) [22] used in the comparison of classifiers. In addition, it presents mathematical notations of the performance metrics. Afterward, this section compares the performance results of the fuzzy-based classifiers, i.e., Fuzzy *k*NN [23], Fuzzy Soft Set Classifier (FSSC) [24], Fuzzy Soft Set Classifier Using Distance-Based Similarity Measure (FussCyier) [25], Hamming Distance-Based Fuzzy Soft Set Classifier (HDFSSC) [26], FPFSCC, FPFSNHC, FPFS-EC, FPFS-AC, FPFS-CMC, FPFS-kNN(P), FPFS-kNN(S), and FPFS-kNN(K), with the performance results of IFPIFSC and the non-fuzzy-based classifiers, i.e., Support Vector Machines (SVM) [27], Decision Trees (DT) [28], Boosting Trees (BT) [29], Random Forests (RF) [30], and Adaptive Boosting (AdaBoost) [31], with those of IFPIFSC. Furthermore, this section performs the statistical evaluations of the performance results using Friedman [32] and Nemenyi [33] tests in a procedure suggested by Demšar [34] and presents the critical diagrams of the Nemenyi test. Furthermore, it compares the classifiers' time complexities using a big O notation. The last section discusses classifiers that can be developed by distance/similarity measures of *ifpifs*-matrices and the need for further research.

2. Preliminaries

This section presents the concept *ifpifs*-matrices [21] and some of its basic properties. Throughout this study, let E be a parameter set and U be an alternative (object) set.

Definition 1 ([15]). *Let μ and ν be two functions from E to [0,1] such that $\mu(x) + \nu(x) \leq 1$, for all $x \in E$. Then, the set $\{(x, \mu(x), \nu(x)) : x \in E\}$ is called an intuitionistic fuzzy set (if-set) over E.*

Here, for all $x \in E$, $\mu(x)$ and $\nu(x)$ are called the membership and non-membership degrees, respectively, and $\pi(x) = 1 - \mu(x) - \nu(x)$ is called the indeterminacy degree of the element $x \in E$. Moreover, for all $x \in E$, $0 \leq \pi(x) \leq 1$ is straightforward. Across the present study, the set of all the *if*-sets over E is denoted by $IF(E)$ and $f \in IF(E)$. For brevity, the notation ${}^{\mu(x)}_{\nu(x)}x$ is used instead of $(x, \mu(x), \nu(x))$. That is, an *if*-set f over E is denoted by $f = \left\{ {}^{\mu(x)}_{\nu(x)}x : x \in E \right\}$.

Definition 2 ([20]). *Let $f \in IF(E)$ and α be a function from f to $IF(U)$. Then, the set*

$$\left\{ \left({}^{\mu(x)}_{\nu(x)}x, \alpha\left({}^{\mu(x)}_{\nu(x)}x \right) \right) : x \in E \right\}$$

being the graphic of α, is called an ifpifs-set parameterized via E over U (or briefly over U).

Hereinafter, the set of all the *ifpifs*-sets over U is denoted by $IFPIFS_E(U)$. Further, in $IFPIFS_E(U)$, since the graph(α) and α generate each other uniquely, the notations are interchangeable. Therefore, if it causes no confusion, we denote an *ifpifs*-set graph(α) by α.

Definition 3 ([21]). *Let $\alpha \in IFPIFS_E(U)$. Then, $[a_{ij}]$ is called ifpifs-matrix of α and defined by*

$$[a_{ij}] := \begin{bmatrix} a_{01} & a_{02} & a_{03} & \cdots & a_{0n} & \cdots \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{0, 1, 2, \cdots\}$ and $j \in \{1, 2, \cdots\}$,

$$a_{ij} := \begin{cases} {}^{\mu(x_j)}_{\nu(x_j)}, & i = 0 \\ \alpha\left({}^{\mu(x_j)}_{\nu(x_j)}x_j \right)(u_i), & i \neq 0 \end{cases}$$

or briefly $a_{ij} := {}^{\mu_{ij}}_{\nu_{ij}}$. Here, if $|U| = m - 1$ and $|E| = n$, then $[a_{ij}]$ is an $m \times n$ ifpifs-matrix.

In this paper, if it causes no confusion, the membership and non-membership functions of $[a_{ij}]$, i.e., μ_{ij} and ν_{ij}, will be represented by μ^a_{ij} and ν^a_{ij}, respectively. Moreover, the set of all the *ifpifs*-matrices parameterized via E over U is denoted by $IFPIFS_E[U]$ and $[a_{ij}], [b_{ij}], [c_{ij}] \in IFPIFS_E[U]$.

Definition 4 ([21]). *Let $[a_{ij}] \in IFPIFS_E[U]$. For all i and j, if $\mu_{ij} = \lambda$ and $\nu_{ij} = \varepsilon$, then $[a_{ij}]$ is called (λ, ε)-ifpifs-matrix and denoted by $\left[\begin{smallmatrix}\lambda\\ \varepsilon\end{smallmatrix}\right]$. Here, $\left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$ and $\left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ are called empty and universal ifpifs-matrices, respectively.*

Definition 5 ([21]). *Let $[a_{ij}], [b_{ij}] \in IFPIFS_E[U]$.*

i. For all i and j, if $\mu^a_{ij} = \mu^b_{ij}$ and $\nu^a_{ij} = \nu^b_{ij}$, then it is said to be $[a_{ij}]$ and $[b_{ij}]$ are equal ifpifs-matrices and denoted by $[a_{ij}] = [b_{ij}]$.
ii. For all i and j, if $\mu^a_{ij} \leq \mu^b_{ij}$ and $\nu^b_{ij} \leq \nu^a_{ij}$, then it is said to be $[a_{ij}]$ is a submatrix of $[b_{ij}]$ and denoted by $[a_{ij}] \tilde{\subseteq} [b_{ij}]$.
iii. If $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ and $[a_{ij}] \neq [b_{ij}]$, then it is said to be $[a_{ij}]$ is a proper submatrix of $[b_{ij}]$ and denoted by $[a_{ij}] \tilde{\subsetneq} [b_{ij}]$.

3. Distance and Similarity Measures of *ifpifs*-Matrices

This section defines metric, quasi-, semi-, and pseudo-metric over $IFPIFS_E[U]$, proposes Minkowski, Euclidean, Hamming, generalized Hausdorff, Hausdorff pseudo-metrics, and their normalized forms, and investigates some of their basic properties. Afterward, the section defines similarity, quasi-, semi-, and pseudo-similarity over $IFPIFS_E[U]$, suggests Minkowski, Euclidean, Hamming, generalized Hausdorff, Hausdorff, Jaccard, Dice, and Cosine pseudo-similarities, and examines some of their basic properties. This section theoretically contributes to the next section in which the advantages of *ifpifs*-matrices are employed in classification problems. In other words, this section provides the advantage of relaying the modeling capability of *ifpifs*-matrices to machine learning via distance and similarity measures defined over $IFPIFS_E[U]$. From now on, let $I_n = \{1, 2, \cdots, n\}$ and $I^*_n = \{0, 1, 2, \cdots, n\}$.

3.1. Distance Measures of ifpifs-Matrices

This subsection first defines metric, quasi-, semi-, and pseudo-metric over $IFPIFS_E[U]$. Let $d : X \times X \to \mathbb{R}$ be a mapping and for all $x, y, z \in X$, D1, D2, D3, D4, and D5 denote the following properties:

D1. $d(x, y) \geq 0$ (Positive semi-definiteness);
D2. $d(x, x) = 0$;
D3. $d(x, y) = 0 \Leftrightarrow x = y$;
D4. $d(x, y) = d(y, x)$ (Symmetry);
D5. $d(x, y) \leq d(x, z) + d(z, y)$ (Triangle inequality).

Definition 6. *Let $d : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ be a mapping. Then,*

i. d is called a quasi-metric iff d satisfies D1, D3, and D5.
ii. d is called a semi-metric iff d satisfies D1, D3, and D4.
iii. d is called a pseudo-metric iff d satisfies D2, D4, and D5.
iv. d is called a metric iff d satisfies D3, D4, and D5.

Secondly, this subsection proposes Minkowski, Euclidean, Hamming, generalized Hausdorff, and Hausdorff pseudo-metrics over $IFPIFS_E[U]$ and their normalized forms and investigates some of their basic properties.

Proposition 1. *Let $p \in \mathbb{Z}^+$. Then, the mapping $d_M^p : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ defined by*

$$d_M^p([a_{ij}],[b_{ij}]) := \left(\frac{1}{2}\sum_{i=1}^{m-1}\sum_{j=1}^{n}\left(\left|\mu_{0j}^a\mu_{ij}^a - \mu_{0j}^b\mu_{ij}^b\right|^p + \left|\nu_{0j}^a\nu_{ij}^a - \nu_{0j}^b\nu_{ij}^b\right|^p + \left|\pi_{0j}^a\pi_{ij}^a - \pi_{0j}^b\pi_{ij}^b\right|^p\right)\right)^{\frac{1}{p}}$$

is a pseudo-metric over $IFPIFS_E[U]$ and referred to as Minkowski pseudo-metric (MPM). Furthermore, the normalized MPM is as follows:

$$\hat{d}_M^p([a_{ij}],[b_{ij}]) := \left(\frac{1}{2(m-1)n}\sum_{i=1}^{m-1}\sum_{j=1}^{n}\left(\left|\mu_{0j}^a\mu_{ij}^a - \mu_{0j}^b\mu_{ij}^b\right|^p + \left|\nu_{0j}^a\nu_{ij}^a - \nu_{0j}^b\nu_{ij}^b\right|^p + \left|\pi_{0j}^a\pi_{ij}^a - \pi_{0j}^b\pi_{ij}^b\right|^p\right)\right)^{\frac{1}{p}}$$

Here, d_M^1 and d_M^2 are called Hamming pseudo-metric (HPM) and Euclidean pseudo-metric (EPM) and denoted by d_H and d_E, respectively. Moreover, $\hat{d}_M^1$ and $\hat{d}_M^2$ are called normalized HPM and normalized EPM and denoted by $\hat{d}_H$ and $\hat{d}_E$, respectively.

Proposition 2. *Let $p \in \mathbb{Z}^+$. Then, the mapping $d_{Hs}^p : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ defined by*

$$d_{Hs}^p([a_{ij}],[b_{ij}]) := \left(\sum_{i=1}^{m-1}\max_{j\in I_n}\left(|\mu_{0j}^a\mu_{ij}^a - \mu_{0j}^b\mu_{ij}^b|^p + |\nu_{0j}^a\nu_{ij}^a - \nu_{0j}^b\nu_{ij}^b|^p + |\pi_{0j}^a\pi_{ij}^a - \pi_{0j}^b\pi_{ij}^b|^p\right)\right)^{\frac{1}{p}}$$

is a pseudo-metric and referred to as generalized Hausdorff pseudo-metric (GHPM). In addition, normalized GHPM is as follows:

$$\hat{d}_{Hs}^p([a_{ij}],[b_{ij}]) := \left(\frac{1}{m-1}\sum_{i=1}^{m-1}\max_{j\in I_n}\left(|\mu_{0j}^a\mu_{ij}^a - \mu_{0j}^b\mu_{ij}^b|^p + |\nu_{0j}^a\nu_{ij}^a - \nu_{0j}^b\nu_{ij}^b|^p + |\pi_{0j}^a\pi_{ij}^a - \pi_{0j}^b\pi_{ij}^b|^p\right)\right)^{\frac{1}{p}}$$

Here, d_{Hs}^1 is called Hausdorff pseudo-metric (HsPM) and denoted by d_{Hs}. Moreover, $\hat{d}_{Hs}^1$ is called normalized HsPM and denoted by $\hat{d}_{Hs}$.

Proposition 3. *Let $p \in \mathbb{Z}^+$ and $[a_{ij}]_{m\times n}, [b_{ij}]_{m\times n}, [c_{ij}]_{m\times n} \in IFPIFS_E[U]$. Then, the following properties are valid.*

i. $d_M^p\left(\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right],\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\right) = d_{Hs}^p\left(\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right],\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\right) = 1,$

ii. $d_M^p([a_{ij}],[b_{ij}]) \leq \sqrt[p]{(m-1)n},$

iii. $d_{Hs}^p([a_{ij}],[b_{ij}]) \leq \sqrt[p]{m-1},$

iv. $[a_{ij}]\tilde{\subseteq}[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow d_M^p([a_{ij}],[b_{ij}]) \leq d_M^p([a_{ij}],[c_{ij}]) \wedge d_M^p([b_{ij}],[c_{ij}]) \leq d_M^p([a_{ij}],[c_{ij}]),$

v. $[a_{ij}]\tilde{\subseteq}[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow d_{Hs}^p([a_{ij}],[b_{ij}]) \leq d_{Hs}^p([a_{ij}],[c_{ij}]) \wedge d_{Hs}^p([b_{ij}],[c_{ij}]) \leq d_{Hs}^p([a_{ij}],[c_{ij}]).$

Remark 1. *The propositions provided in Proposition 3 are also valid for the normalized pseudo-metrics $\hat{d}_M^p$ and $\hat{d}_{Hs}^p$.*

3.2. Similarity Measures of ifpifs-Matrices

This subsection first defines similarity, quasi-, semi-, and pseudo-similarity over $IFPIFS_E[U]$. Let $s : X \times X \to \mathbb{R}$ be a mapping and for all $x, y, z \in X$, S1, S2, S3, and S4 denote the following properties:

S1. $s(x,x) = 1,$

S2. $s(x,y) = 1 \Leftrightarrow x = y,$

S3. $s(x,y) = s(y,x),$

S4. $0 \leq s(x,y) \leq 1.$

Definition 7. *Let $s : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ be a mapping. Then,*

i. s is called a similarity iff d satisfies S2, S3, and S4.
ii. s is called a quasi-similarity iff d satisfies S2 and S4.
iii. s is called a semi-similarity iff d satisfies S2 and S3.
iv. s is called a pseudo-similarity iff d satisfies S1, S3, and S4.

Secondly, this subsection proposes Minkowski, Euclidean, Hamming [35], generalized Hausdorff, and Hausdorff pseudo-similarities over $IFPIFS_E[U]$ using normalized pseudo-metrics of *ifpifs*-matrices provided in Section 3.1.

Proposition 4. *Let $p \in \mathbb{Z}^+$. Then, the mapping $s_M^p : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ defined by*

$$s_M^p([a_{ij}],[b_{ij}]) := 1 - \hat{d}_M^p([a_{ij}],[b_{ij}])$$

is a pseudo-similarity and referred to as Minkowski pseudo-similarity (MPS).

Here, s_M^1 and s_M^2 are called Hamming pseudo-similarity (HPS) [35] and Euclidean pseudo-similarity (EPS) and denoted by s_H and s_E, respectively.

Proposition 5. *Let $p \in \mathbb{Z}^+$. Then, the mapping s_{Hs}^p: $IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ defined by*

$$s_{Hs}^p([a_{ij}],[b_{ij}]) := 1 - \hat{d}_{Hs}^p([a_{ij}],[b_{ij}])$$

is a pseudo-similarity and referred to as generalized Hausdorff pseudo-similarity (GHsPS).

Here, s_{Hs}^1 is called Hausdorff pseudo-similarity (HsPS) and denoted by s_{Hs}. Thirdly, this subsection suggests Jaccard, Dice, and Cosine pseudo-similarities over $IFPIFS_E[U]$.

Proposition 6. *The mapping $s_J : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ defined by*

$$s_J([a_{ij}],[b_{ij}]) := \frac{1}{m-1}\sum_{i=1}^{m-1} \frac{\varepsilon + x_i}{\varepsilon + y_i + z_i - x_i}$$

such that

$$x_i = \sum_{j=1}^{n} \mu_{0j}^a \mu_{ij}^a \mu_{0j}^b \mu_{ij}^b + \nu_{0j}^a \nu_{ij}^a \nu_{0j}^b \nu_{ij}^b + \pi_{0j}^a \pi_{ij}^a \pi_{0j}^b \pi_{ij}^b$$

$$y_i = \sum_{j=1}^{n} \left(\mu_{0j}^a \mu_{ij}^a\right)^2 + \left(\nu_{0j}^a \nu_{ij}^a\right)^2 + \left(\pi_{0j}^a \pi_{ij}^a\right)^2$$

and

$$z_i = \sum_{j=1}^{n} \left(\mu_{0j}^b \mu_{ij}^b\right)^2 + \left(\nu_{0j}^b \nu_{ij}^b\right)^2 + \left(\pi_{0j}^b \pi_{ij}^b\right)^2$$

is a pseudo-similarity and referred to as Jaccard pseudo-similarity (JPS). Here, $\varepsilon \ll 1$ is a positive constant, e.g., $\varepsilon = 0.0001$.

Proof. Let $[a_{ij}],[b_{ij}] \in IFPIFS_E[U]$. It is clear that s_J satisfies the conditions S1 and S3. Then, it is sufficient to prove the condition S4. For $i \in I_{m-1}$ and for all $j \in I_n$,

$$\begin{aligned} 0 &\leq \mu_{0j}^a \mu_{ij}^a \mu_{0j}^b \mu_{ij}^b + \nu_{0j}^a \nu_{ij}^a \nu_{0j}^b \nu_{ij}^b + \pi_{0j}^a \pi_{ij}^a \pi_{0j}^b \pi_{ij}^b \\ &\leq \left(\mu_{0j}^a \mu_{ij}^a\right)^2 + \left(\nu_{0j}^a \nu_{ij}^a\right)^2 + \left(\pi_{0j}^a \pi_{ij}^a\right)^2 + \left(\mu_{0j}^b \mu_{ij}^b\right)^2 + \left(\nu_{0j}^b \nu_{ij}^b\right)^2 + \left(\pi_{0j}^b \pi_{ij}^b\right)^2 \\ &\quad - \left(\mu_{0j}^a \mu_{ij}^a \mu_{0j}^b \mu_{ij}^b + \nu_{0j}^a \nu_{ij}^a \nu_{0j}^b \nu_{ij}^b + \pi_{0j}^a \pi_{ij}^a \pi_{0j}^b \pi_{ij}^b\right) \end{aligned}$$

because

$$0 \leq \left(\mu^a_{0j}\mu^a_{ij} - \mu^b_{0j}\mu^b_{ij}\right)^2 + \left(\nu^a_{0j}\nu^a_{ij} - \nu^b_{0j}\nu^b_{ij}\right)^2 + \left(\pi^a_{0j}\pi^a_{ij} - \pi^b_{0j}\pi^b_{ij}\right)^2$$

Therefore,

$$0 \leq \varepsilon + x_i \leq \varepsilon + y_i + z_i - x_i$$

Hence,

$$0 \leq \frac{\varepsilon + x_i}{\varepsilon + y_i + z_i - x_i} \leq 1$$

Then,

$$\begin{aligned}
\tfrac{1}{m-1}\sum_{i=1}^{m-1} 0 &\leq s_J([a_{ij}],[b_{ij}]) \leq \tfrac{1}{m-1}\sum_{i=1}^{m-1} 1 \\
0 &\leq s_J([a_{ij}],[b_{ij}]) \leq \tfrac{1}{m-1}(m-1) \\
0 &\leq s_J([a_{ij}],[b_{ij}]) \leq 1
\end{aligned}$$

□

Proposition 7. *The mapping* $s_D : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ *defined by*

$$s_D([a_{ij}],[b_{ij}]) := \frac{1}{m-1}\sum_{i=1}^{m-1} \frac{\varepsilon + 2x_i}{\varepsilon + y_i + z_i}$$

such that

$$x_i = \sum_{j=1}^{n} \mu^a_{0j}\mu^a_{ij}\mu^b_{0j}\mu^b_{ij} + \nu^a_{0j}\nu^a_{ij}\nu^b_{0j}\nu^b_{ij} + \pi^a_{0j}\pi^a_{ij}\pi^b_{0j}\pi^b_{ij}$$

$$y_i = \sum_{j=1}^{n} \left(\mu^a_{0j}\mu^a_{ij}\right)^2 + \left(\nu^a_{0j}\nu^a_{ij}\right)^2 + \left(\pi^a_{0j}\pi^a_{ij}\right)^2$$

and

$$z_i = \sum_{j=1}^{n} \left(\mu^b_{0j}\mu^b_{ij}\right)^2 + \left(\nu^b_{0j}\nu^b_{ij}\right)^2 + \left(\pi^b_{0j}\pi^b_{ij}\right)^2$$

is a pseudo-similarity and referred to as Dice pseudo-similarity (DPS). Here, $\varepsilon \ll 1$ *is a positive constant, e.g.,* $\varepsilon = 0.0001$.

Proof. Let $[a_{ij}], [b_{ij}] \in IFPIFS_E[U]$. It is clear that s_D satisfies the conditions S1 and S3. Then, it is sufficient to prove the condition S4. For $i \in I_{m-1}$ and for all $j \in I_n$, since

$$\begin{aligned}
0 &\leq \left(\mu^a_{0j}\mu^a_{ij} - \mu^b_{0j}\mu^b_{ij}\right)^2 + \left(\nu^a_{0j}\nu^a_{ij} - \nu^b_{0j}\nu^b_{ij}\right)^2 + \left(\pi^a_{0j}\pi^a_{ij} - \pi^b_{0j}\pi^b_{ij}\right)^2 \\
&= \left(\mu^a_{0j}\mu^a_{ij}\right)^2 + \left(\nu^a_{0j}\nu^a_{ij}\right)^2 + \left(\pi^a_{0j}\pi^a_{ij}\right)^2 + \left(\mu^b_{0j}\mu^b_{ij}\right)^2 + \left(\nu^b_{0j}\nu^b_{ij}\right)^2 + \left(\pi^b_{0j}\pi^b_{ij}\right)^2 \\
&\quad -2\left(\mu^a_{0j}\mu^a_{ij}\mu^b_{0j}\mu^b_{ij} + \nu^a_{0j}\nu^a_{ij}\nu^b_{0j}\nu^b_{ij} + \pi^a_{0j}\pi^a_{ij}\pi^b_{0j}\pi^b_{ij}\right)
\end{aligned}$$

then

$$0 \leq \varepsilon + 2x_i \leq \varepsilon + y_i + z_i$$

Hence,

$$0 \leq \frac{\varepsilon + 2x_i}{\varepsilon + y_i + z_i} \leq 1$$

Then,

$$\begin{aligned}
\frac{1}{m-1}\sum_{i=1}^{m-1} 0 &\leq s_D([a_{ij}],[b_{ij}]) \leq \frac{1}{m-1}\sum_{i=1}^{m-1} 1 \\
0 &\leq s_D([a_{ij}],[b_{ij}]) \leq \frac{1}{m-1}(m-1) \\
0 &\leq s_D([a_{ij}],[b_{ij}]) \leq 1
\end{aligned}$$

□

Proposition 8. *The mapping $s_C : IFPIFS_E[U] \times IFPIFS_E[U] \to \mathbb{R}$ defined by*

$$s_C([a_{ij}],[b_{ij}]) := \frac{1}{m-1}\sum_{i=1}^{m-1} \frac{\varepsilon + x_i}{\varepsilon + \sqrt{y_i}\sqrt{z_i}}$$

such that

$$x_i = \sum_{j=1}^{n} \mu_{0j}^{a}\mu_{ij}^{a}\mu_{0j}^{b}\mu_{ij}^{b} + \nu_{0j}^{a}\nu_{ij}^{a}\nu_{0j}^{b}\nu_{ij}^{b} + \pi_{0j}^{a}\pi_{ij}^{a}\pi_{0j}^{b}\pi_{ij}^{b}$$

$$y_i = \sum_{j=1}^{n} \left(\mu_{0j}^{a}\mu_{ij}^{a}\right)^2 + \left(\nu_{0j}^{a}\nu_{ij}^{a}\right)^2 + \left(\pi_{0j}^{a}\pi_{ij}^{a}\right)^2$$

and

$$z_i = \sum_{j=1}^{n} \left(\mu_{0j}^{b}\mu_{ij}^{b}\right)^2 + \left(\nu_{0j}^{b}\nu_{ij}^{b}\right)^2 + \left(\pi_{0j}^{b}\pi_{ij}^{b}\right)^2$$

is a pseudo-similarity and referred to as Cosine pseudo-similarity (CPS). Here, $\varepsilon \ll 1$ is a positive constant, e.g., $\varepsilon = 0.0001$.

Proposition 9. *Let $p \in \mathbb{Z}^+$ and $[a_{ij}]_{m\times n}, [b_{ij}]_{m\times n}, [c_{ij}]_{m\times n} \in IFPIFS_E[U]$. Then, the following properties are valid.*

i. $s_M^p\left(\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right],\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\right) = s_{Hs}^p\left(\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right],\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\right) = s_J\left(\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right],\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\right) = s_D\left(\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right],\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\right) = s_C\left(\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right],\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]\right) = 0,$

ii. $[a_{ij}]\tilde{\subseteq}[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow s_M^p([a_{ij}],[c_{ij}]) \leq s_M^p([a_{ij}],[b_{ij}]) \wedge s_M^p([a_{ij}],[c_{ij}]) \leq s_M^p([b_{ij}],[c_{ij}]),$

iii. $[a_{ij}]\tilde{\subseteq}[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow s_{Hs}^p([a_{ij}],[c_{ij}]) \leq s_{Hs}^p([a_{ij}],[b_{ij}]) \wedge s_{Hs}^p([a_{ij}],[c_{ij}]) \leq s_{Hs}^p([b_{ij}],[c_{ij}]),$

iv. $[a_{ij}]\tilde{\subseteq}[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow s_J([a_{ij}],[c_{ij}]) \leq s_J([a_{ij}],[b_{ij}]) \wedge s_J([a_{ij}],[c_{ij}]) \leq s_J([b_{ij}],[c_{ij}]),$

v. $[a_{ij}]\tilde{\subseteq}[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow s_D([a_{ij}],[c_{ij}]) \leq s_D([a_{ij}],[b_{ij}]) \wedge s_D([a_{ij}],[c_{ij}]) \leq s_D([a_{ij}],[c_{ij}]),$

vi. $[a_{ij}]\tilde{\subseteq}[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow s_C([a_{ij}],[c_{ij}]) \leq s_C([a_{ij}],[b_{ij}]) \wedge s_C([a_{ij}],[c_{ij}]) \leq s_C([b_{ij}],[c_{ij}]).$

4. Proposed Classifier (IFPIFSC)

This section presents the basic mathematical notations to be needed for the proposed classifier based on *ifpifs*-matrices. Throughout the present study, let $D = [d_{ij}]_{m\times(n+1)}$ stand for a data matrix whose last column consists of the data's labels, where m and n represent the samples' and parameters' numbers in the data matrix, respectively. $(D_{train})_{m_1\times n}$, $C_{m_1\times 1}$, and $(D_{test})_{m_2\times n}$ stand for a training matrix, class matrix of the training matrix, and the testing matrix attained from the data matrix D, respectively, such that $m_1 + m_2 = m$. Let $U_{k\times 1}$ be a matrix consisting of unique class labels of $C_{m_1\times 1}$. $D_{i-train}$ and D_{i-test} denote i-th rows of D_{train} and D_{test}, respectively. Similarly, $D_{train-j}$ and D_{test-j} denote j-th rows of D_{train} and D_{test}, respectively. Moreover, $T'_{m_2\times 1}$ represents predicted class labels of the testing samples.

Definition 8. *Let $x, y \in \mathbb{R}^n$. Then, the function $P : \mathbb{R}^n \times \mathbb{R}^n \to [-1, 1]$ defined by*

$$P(x,y) := \frac{n\sum_{j=1}^{n} x_j y_j - \left(\sum_{j=1}^{n} x_j\right)\left(\sum_{j=1}^{n} y_j\right)}{\sqrt{\left[n\sum_{j=1}^{n} x_j^2 - \left(\sum_{j=1}^{n} x_j\right)^2\right]\left[n\sum_{j=1}^{n} y_j^2 - \left(\sum_{j=1}^{n} y_j\right)^2\right]}}$$

is called the Pearson correlation coefficient between x and y.

Definition 9. *Let $x \in \mathbb{R}^n$ and $j \in I_n$. Then, the vector $\hat{x} \in \mathbb{R}^n$ defined by*

$$\hat{x}_j := \begin{cases} \frac{x_j - \min_{k\in I_n}\{x_k\}}{\max_{k\in I_n}\{x_k\} - \min_{k\in I_n}\{x_k\}}, & \max_{k\in I_n}\{x_k\} \neq \min_{k\in I_n}\{x_k\} \\ 1, & \max_{k\in I_n}\{x_k\} = \min_{k\in I_n}\{x_k\} \end{cases}$$

is called normalizing vector of x.

Definition 10. *Let $D = [d_{ij}]_{m\times(n+1)}$ be a data matrix, $i \in I_m$, and $j \in I_n$. Then, the matrix $\tilde{D} = [\tilde{d}_{ij}]_{m\times n}$ defined by*

$$\tilde{d}_{ij} := \begin{cases} \frac{d_{ij} - \min_{k\in I_m}\{d_{kj}\}}{\max_{k\in I_m}\{d_{kj}\} - \min_{k\in I_m}\{d_{kj}\}}, & \max_{k\in I_m}\{d_{kj}\} \neq \min_{k\in I_m}\{d_{kj}\} \\ 1, & \max_{k\in I_m}\{d_{kj}\} = \min_{k\in I_m}\{d_{kj}\} \end{cases}$$

is called column normalized matrix (feature-fuzzification matrix) of D.

Definition 11. *Let $(D_{train})_{m_1\times n}$ be a training matrix obtained from $D = [d_{ij}]_{m\times(n+1)}$, $i \in I_{m_1}$, and $j \in I_n$. Then, the matrix $\tilde{D}_{train} = [\tilde{d}_{ij-train}]_{m_1\times n}$ defined by*

$$\tilde{d}_{ij-train} := \begin{cases} \frac{d_{ij-train} - \min_{k\in I_m}\{d_{kj}\}}{\max_{k\in I_m}\{d_{kj}\} - \min_{k\in I_m}\{d_{kj}\}}, & \max_{k\in I_m}\{d_{kj}\} \neq \min_{k\in I_m}\{d_{kj}\} \\ 1, & \max_{k\in I_m}\{d_{kj}\} = \min_{k\in I_m}\{d_{kj}\} \end{cases}$$

is called column normalized matrix (feature-fuzzification matrix) of D_{train}.

Definition 12. *Let $(D_{test})_{m_2\times n}$ be a testing matrix obtained from $D = [d_{ij}]_{m\times(n+1)}$, $i \in I_{m_2}$, and $j \in I_n$. Then, the matrix $\tilde{D}_{test} = [\tilde{d}_{ij-test}]_{m_1\times n}$ defined by*

$$\tilde{d}_{ij-test} := \begin{cases} \frac{d_{ij-test} - \min_{k\in I_m}\{d_{kj}\}}{\max_{k\in I_m}\{d_{kj}\} - \min_{k\in I_m}\{d_{kj}\}}, & \max_{k\in I_m}\{d_{kj}\} \neq \min_{k\in I_m}\{d_{kj}\} \\ 1, & \max_{k\in I_m}\{d_{kj}\} = \min_{k\in I_m}\{d_{kj}\} \end{cases}$$

is called column normalized matrix (feature-fuzzification matrix) of D_{test}.

Definition 13 ([35])**.** *Let $D_{train} = [d_{ij-train}]_{m_1\times n}$ and $C_{m_1\times n}$ be a training matrix and its class matrix obtained from a data matrix $D = [d_{ij}]_{m\times(n+1)}$, respectively. Then, the matrix $ifw^{\lambda P}_{D_{train}} = \begin{bmatrix} \mu^{\lambda P}_{1j} \\ \nu^{\lambda P}_{1j} \end{bmatrix}_{1\times n}$ is called intuitionistic fuzzification weight matrix based on Pearson correlation coefficient of D_{train} and defined by*

$$\mu^{\lambda P}_{1j} := 1 - (1 - |P(D_{train-j}, C)|)^{\lambda}$$

and

$$\nu_{1j}^{\lambda P} := (1 - |P(D_{train-j}, C)|)^{\lambda(\lambda+1)}$$

such that $j \in I_n$ *and* $\lambda \in [0, \infty)$.

Definition 14 ([35]). *Let* $\tilde{D}_{train} = [\tilde{d}_{ij-train}]_{m_1 \times n}$ *be a column normalized matrix of a matrix* $(D_{train})_{m_1 \times n}$. *Then, the matrix* $\tilde{\tilde{D}}^{\lambda}_{train} = [\tilde{\tilde{d}}^{\lambda}_{train-ij}] = \begin{bmatrix} \mu^{\tilde{\tilde{D}}^{\lambda}}_{ij-train} \\ \nu^{\tilde{\tilde{D}}^{\lambda}}_{ij-train} \end{bmatrix}_{m_1 \times n}$ *is called intuitionistic fuzzification of* $\tilde{D}_{train}$ *and defined by*

$$\mu^{\tilde{\tilde{D}}^{\lambda}}_{ij-train} := 1 - (1 - \tilde{d}_{ij-train})^{\lambda}$$

and

$$\nu^{\tilde{\tilde{D}}^{\lambda}}_{ij-train} := (1 - \tilde{d}_{ij-train})^{\lambda(\lambda+1)}$$

such that $i \in I_{m_1}$, $j \in I_n$, *and* $\lambda \in [0, \infty)$.

Definition 15 ([35]). *Let* $\tilde{D}_{test} = [\tilde{d}_{ij-test}]_{m_2 \times n}$ *be a column normalized matrix of a matrix* $(D_{test})_{m_2 \times n}$. *Then, the matrix* $\tilde{\tilde{D}}^{\lambda}_{test} = [\tilde{\tilde{d}}^{\lambda}_{test-ij}] = \begin{bmatrix} \mu^{\tilde{\tilde{D}}^{\lambda}}_{ij-test} \\ \nu^{\tilde{\tilde{D}}^{\lambda}}_{ij-test} \end{bmatrix}_{m_2 \times n}$ *is called intuitionistic fuzzification of* $\tilde{D}_{test}$ *and defined by*

$$\mu^{\tilde{\tilde{D}}^{\lambda}}_{ij-test} := 1 - (1 - \tilde{d}_{ij-test})^{\lambda}$$

and

$$\nu^{\tilde{\tilde{D}}^{\lambda}}_{ij-test} := (1 - \tilde{d}_{ij-test})^{\lambda(\lambda+1)}$$

such that $i \in I_{m_2}$, $j \in I_n$, *and* $\lambda \in [0, \infty)$.

Definition 16 ([35]). *Let* $(\tilde{D}_{train})_{m_1 \times n}$ *be a column normalized matrix of a matrix* $(D_{train})_{m_1 \times n}$ *and* $\tilde{\tilde{D}}^{\lambda}_{train} = [\tilde{\tilde{d}}^{\lambda}_{train-ij}] = \begin{bmatrix} \mu^{\tilde{\tilde{D}}^{\lambda}}_{ij-train} \\ \nu^{\tilde{\tilde{D}}^{\lambda}}_{ij-train} \end{bmatrix}_{m_1 \times n}$ *be the intuitionistic fuzzification of* $\tilde{D}_{train}$. *Then, the ifpifs-matrix* $\left[b^{\tilde{\tilde{D}}^{\lambda}_{k-train}}_{ij} \right]_{2 \times n}$ *is called the training ifpifs-matrix obtained by k-th row of* $\tilde{\tilde{D}}^{\lambda}_{train}$ *and* $ifw^{\lambda P}_{D_{train}}$ *and defined by*

$$b^{\tilde{\tilde{D}}^{\lambda}_{k-train}}_{0j} := \begin{smallmatrix} \mu^{\lambda P}_{1j} \\ \nu^{\lambda P}_{1j} \end{smallmatrix} \text{ and } b^{\tilde{\tilde{D}}^{\lambda}_{k-train}}_{1j} := \begin{smallmatrix} \mu^{\tilde{\tilde{D}}^{\lambda}}_{kj-train} \\ \nu^{\tilde{\tilde{D}}^{\lambda}}_{kj-train} \end{smallmatrix}$$

such that $k \in I_{m_1}$ *and* $j \in I_n$.

Definition 17 ([35]). *Let* $(\tilde{D}_{test})_{m_2 \times n}$ *be a column normalized matrix of a matrix* $(D_{test})_{m_2 \times n}$ *and* $\tilde{\tilde{D}}^{\lambda}_{test} = [\tilde{\tilde{d}}^{\lambda}_{test-ij}] = \begin{bmatrix} \mu^{\tilde{\tilde{D}}^{\lambda}}_{ij-test} \\ \nu^{\tilde{\tilde{D}}^{\lambda}}_{ij-test} \end{bmatrix}_{m_2 \times n}$ *be the intuitionistic fuzzification of* $\tilde{D}_{test}$. *Then, the ifpifs-matrix* $\left[a^{\tilde{\tilde{D}}^{\lambda}_{k-test}}_{ij} \right]_{2 \times n}$ *is called the testing ifpifs-matrix obtained by k-th row of* $\tilde{\tilde{D}}^{\lambda}_{test}$ *and* $ifw^{\lambda P}_{D_{train}}$ *and defined by*

$$a^{\tilde{\tilde{D}}^{\lambda}_{k-test}}_{0j} := \begin{smallmatrix} \mu^{\lambda P}_{1j} \\ \nu^{\lambda P}_{1j} \end{smallmatrix} \text{ and } a^{\tilde{\tilde{D}}^{\lambda}_{k-test}}_{1j} := \begin{smallmatrix} \mu^{\tilde{\tilde{D}}^{\lambda}}_{kj-test} \\ \nu^{\tilde{\tilde{D}}^{\lambda}}_{kj-test} \end{smallmatrix}$$

such that $k \in I_{m_1}$ *and* $j \in I_n$.

Afterward, this section proposes a new classifier named IFPIFSC. This classifier utilizes Definition 13 to attain a parameter effect-based feature weight on classification. It then built the training *ifpifs*-matrix and the testing *ifpifs*-matrix using Definitions 11, 12, and 14–17. Later, employing HPS, EPS, MPS, HsPS, JPS, and CPS, a matrix of similarity values of the testing *ifpifs*-matrix to each training *ifpifs*-matrix is obtained. For each pseudo-similarity, the class of the training sample with the highest similarity is found, and the class with the highest frequency value is determined and assigned to the test sample. Similarly, this procedure repeats for all test samples. Lastly, the predicted class matrix is generated for the test data. IFPIFSC's flowchart (Figure 1) and algorithm steps (Algorithm 1) are as follows:

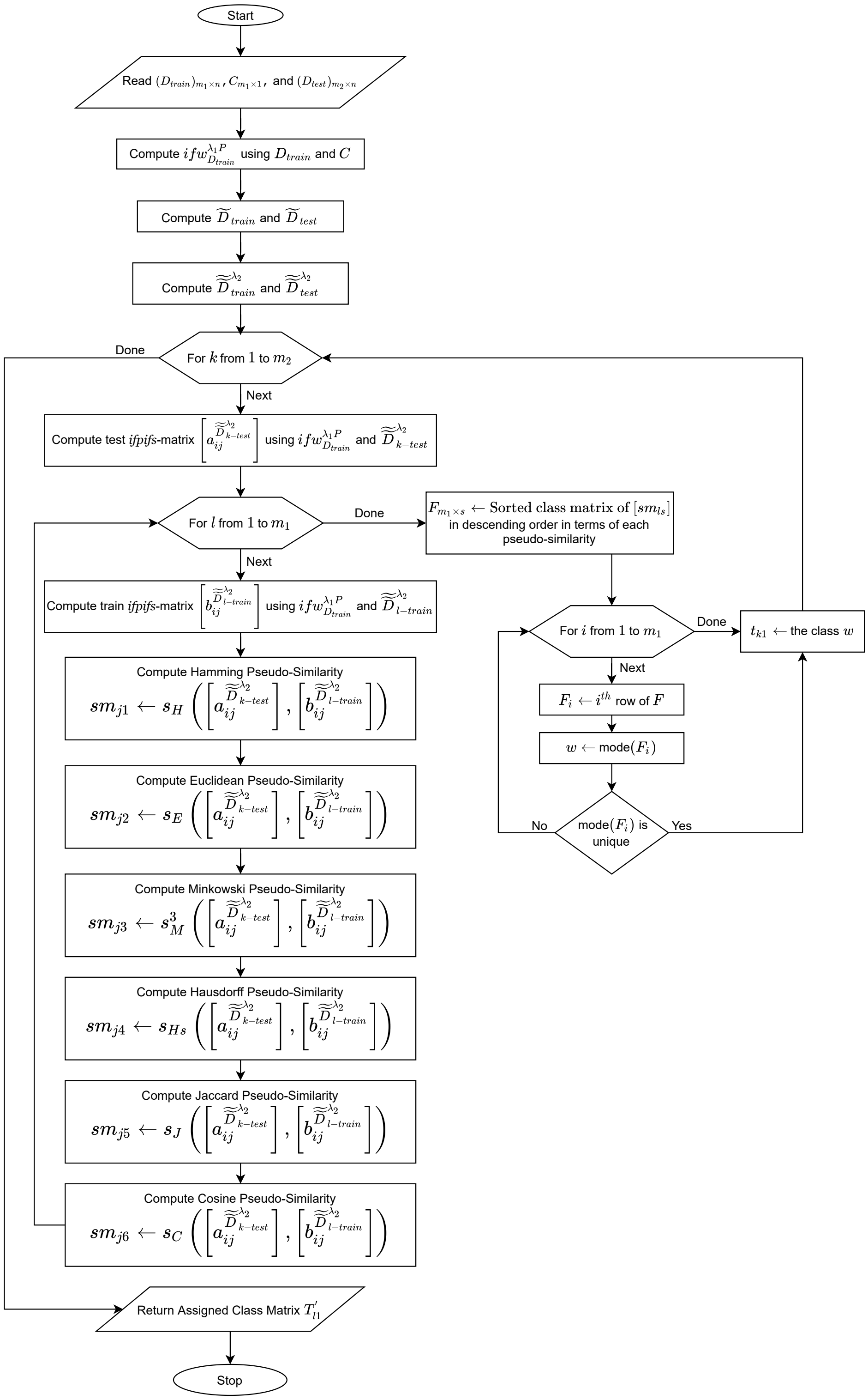

Figure 1. IFPIFSC's flowchart.

Algorithm 1 IFPIFSC's pseudocode

Input: $(D_{train})_{m_1\times n}$, $C_{m_1\times 1}$, $(D_{test})_{m_2\times n}$, λ_1, and λ_2

Output: $T_{m_2\times 1}$

procedure IFPIFSC(D_{train}, C, D_{test}, λ_1, λ_2)
    Compute $ifw^{\lambda_1 P}_{D_{train}}$ using D_{train} and C
    Compute feature fuzzification of D_{train} and D_{test}, namely $\tilde{D}_{train}$ and $\tilde{D}_{test}$
    Compute feature intuitionistic fuzzification of $\tilde{D}_{train}$ and $\tilde{D}_{test}$, namely $\tilde{\tilde{D}}^{\lambda_2}_{train}$ and $\tilde{\tilde{D}}^{\lambda_2}_{test}$
    for k from 1 to m_2 **do**
        Compute the testing *ifpifs*-matrix $\left[a_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-test}}\right]_{2\times n}$ using $ifw^{\lambda_1 P}_{D_{train}}$ and $\tilde{\tilde{D}}^{\lambda_2}_{k-test}$
        for l from 1 to m_1 **do**
            Compute the training *ifpifs*-matrix $\left[b_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-train}}\right]_{2\times n}$ using $ifw^{\lambda_1 P}_{D_{train}}$ and $\tilde{\tilde{D}}^{\lambda_2}_{l-train}$
            $sm_{l1} \leftarrow s_H\left(\left[a_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-test}}\right], \left[b_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-train}}\right]\right)$
            $sm_{l2} \leftarrow s_E\left(\left[a_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-test}}\right], \left[b_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-train}}\right]\right)$
            $sm_{l3} \leftarrow s_M^3\left(\left[a_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-test}}\right], \left[b_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-train}}\right]\right)$
            $sm_{l4} \leftarrow s_{Hs}\left(\left[a_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-test}}\right], \left[b_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-train}}\right]\right)$
            $sm_{l5} \leftarrow s_J\left(\left[a_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-test}}\right], \left[b_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-train}}\right]\right)$
            $sm_{l6} \leftarrow s_C\left(\left[a_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-test}}\right], \left[b_{ij}^{\tilde{\tilde{D}}^{\lambda_2}_{k-train}}\right]\right)$
        end for
        $F_{m_1\times s} \leftarrow$ Sorted class matrix of $[sm_{ls}]$ in descending order in terms of each pseudo-similarity
        for i from 1 to m_1 **do**
            $F_i \leftarrow i$-th row of F
            $w \leftarrow \text{mode}(F_i)$
            if $\text{mode}(F_i)$ is unique **then**
                break
            end if
        end for
        $t_{k1} \leftarrow w$
    end for
    return $T'_{m_2\times 1}$
end procedure

5. Simulation and Performance Comparison

The present section provides the details of the 20 datasets in the UCI-MLR [22] for the classification task. It then presents five performance metrics to be used for performance comparison. Afterward, this section executes a simulation to demonstrate that IFPIFSC exhibits a better-classifying performance than Fuzzy kNN [23], FSSC [24], FussCyier [25], HDFSSC [26], FPFSCC [10], FPFSNHC [9], FPFS-EC [11], FPFS-AC [13], FPFS-CMC [12], FPFS-kNN(P) [14], FPFS-kNN(S) [14], FPFS-kNN(K) [14], SVM [27], DT [28], BT [29], RF [30], and AdaBoost [31] do. Moreover, it performs statistical analyzes of the simulation results employing the Friedman test [32], a non-parametric test, and the Nemenyi test [33], a post hoc test. Finally, this section provides the time complexities of the compared classifiers in compliance with big O notation.

5.1. UCI Datasets and Features

This subsection presents the properties of the following datasets [22], used in the simulation, in Table 1: "Zoo", "Breast Tissue", "Teaching Assistant Evaluation", "Wine", "Parkinsons[sic] ", "Sonar", "Seeds", "Parkinson Acoustic", "Ecoli", "Leaf", "Ionosphere", "Libras Movement", "Dermatology", "Breast Cancer Wisconsin", "HCV Data", "Parkinson's Disease Classification", "Mice Protein Expression", "Semeion Handwritten Digit", "Car Evaluation", and "Wireless Indoor Localization".

Table 1. Descriptions of UCI datasets.

No.	Name	#Instance	#Attribute	#Class	Balanced/Imbalanced
1	Zoo	101	16	7	Imbalanced
2	Breast Tissue	106	9	6	Imbalanced
3	Teaching Assistant Evaluation	151	5	3	Imbalanced
4	Wine	178	13	3	Imbalanced
5	Parkinsons[sic]	195	22	2	Imbalanced
6	Sonar	208	60	2	Imbalanced
7	Seeds	210	7	3	Balanced
8	Parkinson Acoustic	240	46	2	Balanced
9	Ecoli	336	7	8	Imbalanced
10	Leaf	340	14	36	Imbalanced
11	Ionosphere	351	34	2	Imbalanced
12	Libras Movement	360	90	15	Balanced
13	Dermatology	366	34	6	Imbalanced
14	Breast Cancer Wisconsin	569	30	2	Imbalanced
15	HCV Data	589	12	5	Imbalanced
16	Parkinson's Disease Classification	756	754	2	Imbalanced
17	Mice Protein Expression	1077	72	8	Imbalanced
18	Semeion Handwritten Digit	1593	265	2	Imbalanced
19	Car Evaluation	1728	6	4	Imbalanced
20	Wireless Indoor Localization	2000	7	4	Balanced

stands for "the number of".

5.2. Performance Metrics

This subsection provides the mathematical notations of five performance metrics [36–38], i.e., accuracy (Acc), precision (Pre), recall (Rec), macro F-score (MacF), and micro F-score (MicF), to compare the aforementioned classifiers. Let $D_{test} = \{x_1, x_2, \ldots, x_n\}$, $T = \{t_1, t_2, \ldots, t_n\}$, $T' = \{t'_1, t'_2, \ldots, t'_n\}$, and l be n samples to be classified, ground truth class sets of the samples, prediction class sets of the samples, and the number of the class of the samples, respectively.

$$\begin{aligned}
\text{Acc}(T,T') &:= \tfrac{1}{l}\sum_{i=1}^{l} \tfrac{TP_i+TN_i}{TP_i+TN_i+FP_i+FN_i} \\
\text{Pre}(T,T') &:= \tfrac{1}{l}\sum_{i=1}^{l} \tfrac{TP_i}{TP_i+FP_i} \\
\text{Rec}(T,T') &:= \tfrac{1}{l}\sum_{i=1}^{l} \tfrac{TP_i}{TP_i+FN_i} \\
\text{MacF}(T,T') &:= \tfrac{1}{l}\sum_{i=1}^{l} \tfrac{2TP_i}{2TP_i+FP_i+FN_i} \\
\text{MicF}(T,T') &:= \tfrac{2\sum_{i=1}^{l} TP_i}{2\sum_{i=1}^{l} TP_i+\sum_{i=1}^{l} FP_i+\sum_{i=1}^{l} FN_i}
\end{aligned}$$

where TP_i, TN_i, FP_i, and FN_i are the number of true positive, true negative, false positive, and false negative, for the class i, respectively, and their mathematical notations are as follows:

$$\begin{aligned}
TP_i &:= \left|\left\{x_k | i \in T_k \wedge i \in T'_k, 1 \le k \le l\right\}\right| \\
TN_i &:= \left|\left\{x_t | i \notin T_k \wedge i \notin T'_k, 1 \le k \le l\right\}\right| \\
FP_i &:= \left|\left\{x_t | i \notin T_k \wedge i \in T'_k, 1 \le k \le l\right\}\right| \\
FN_i &:= \left|\left\{x_t | i \in T_k \wedge i \notin T'_k, 1 \le k \le l\right\}\right|
\end{aligned}$$

Here, the notation $|.|$ denotes the cardinality of a set.

5.3. Simulation Results

This subsection compares IFPIFSC with the state-of-the-art and well-known classifiers rely on fuzzy and soft sets, i.e., Fuzzy 3NN, FussCyier, FSSC, HDFSSC, FPFSCC, FPFSNHC, FPFS-EC, FPFS-AC, FPFS-CMC, FPFS-3NN(P), FPFS-3NN(S), and FPFS-3NN(K), and other well-known classifiers, i.e., DT, SVM, BT, RF, and AdaBoost, by utilizing MATLAB R2021b and a laptop with I(R) Core(TM) I5-3230M CPU @ 2.60 GHz and 16 GB RAM. Here, the mean performance results of the classifiers are obtained by random 10 independent runs based on the 5-fold cross-validation [38,39]. In each cross-validation, the relevant dataset is randomly split into five parts, and four of the parts are used for training and the other for testing (for more details about k-fold cross-validation, see [39]). Table 2 provides the average Acc, Pre, Rec, MacF, and MicF results of IFPIFSC, Fuzzy 3NN, FSSC, FussCyier, HDFSSC, FPFSCC, FPFSNHC, FPFS-EC, FPFS-AC, FPFS-CMC, FPFS-3NN(P), FPFS-3NN(S), and FPFS-3NN(K) for the datasets.

Table 2. Simulation results of the fuzzy-based classifiers.

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Zoo	Fuzzy 3NN	97.63 ± 1.42	90.41 ± 7.22	84.13 ± 10.2	92.05 ± 5.77	91.77 ± 4.98
	FSSC	97.97 ± 1.32	90.03 ± 9.13	86.56 ± 9.46	93.25 ± 4.92	93.06 ± 4.51
	FussCyier	97.74 ± 1.42	89.39 ± 9.23	86.27 ± 9.46	92.68 ± 5.21	92.26 ± 4.85
	HDFSSC	98.29 ± 1.4	91.72 ± 8.15	87.45 ± 10.93	93.48 ± 5.2	94.15 ± 4.79
	FPFSCC	97.17 ± 2.13	88.27 ± 10.11	82.05 ± 12.41	89.22 ± 7.87	90.27 ± 7.49
	FPFSNHC	98.29 ± 1.43	92 ± 8.53	87.17 ± 11.35	93.26 ± 5.81	94.15 ± 4.9
	FPFS-EC	**98.85 ± 1.12**	**94.34 ± 6.98**	89.86 ± 10.24	**96.6 ± 4.17**	**96.04 ± 3.88**
	FPFS-AC	98.36 ± 1.3	91.66 ± 8.15	85.9 ± 10.94	94.94 ± 5.42	94.35 ± 4.49
	FPFS-CMC	98.73 ± 1.48	93.81 ± 8.43	89.19 ± 12.28	96.31 ± 5.2	95.64 ± 5.08
	FPFS-3NN(P)	98.22 ± 1.29	92.03 ± 7.25	86.67 ± 10.38	93.17 ± 5.13	93.87 ± 4.51
	FPFS-3NN(S)	98.25 ± 1.26	92.35 ± 6.81	87.1 ± 10.18	93.23 ± 5.27	93.97 ± 4.38
	FPFS-3NN(K)	98.25 ± 1.26	92.35 ± 6.81	87.1 ± 10.18	93.23 ± 5.27	93.97 ± 4.38
	IFPIFSC	98.65 ± 1.23	92.79 ± 6.53	**89.92 ± 8.2**	96.31 ± 3.69	95.35 ± 3.38

Table 2. *Cont.*

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Breast Tissue	Fuzzy 3NN	84.37 ± 2.73	56.35 ± 9.71	51.64 ± 8.92	57.4 ± 7.04	53.1 ± 8.18
	FSSC	87.83 ± 2.88	64.48 ± 10.14	61.95 ± 8.99	66.11 ± 7.27	63.48 ± 8.65
	FussCyier	87.19 ± 2.97	64.15 ± 9.11	60.34 ± 9.26	64.79 ± 6.92	61.58 ± 8.91
	HDFSSC	87.73 ± 3	67.57 ± 9.55	62.07 ± 9.08	64.47 ± 8.27	63.2 ± 9
	FPFSCC	87.29 ± 2.65	63.77 ± 10.12	60.17 ± 8.76	67.03 ± 9.22	61.87 ± 7.95
	FPFSNHC	87.89 ± 3.23	66.78 ± 10.22	62.41 ± 10.49	66.32 ± 7.89	63.66 ± 9.69
	FPFS-EC	88.11 ± 2.74	65.95 ± 7.98	63.15 ± 8.84	70.24 ± 8.51	64.33 ± 8.23
	FPFS-AC	89.58 ± 2.66	69.54 ± 8.57	68.05 ± 8.31	71.32 ± 7.91	68.75 ± 7.98
	FPFS-CMC	87.82 ± 2.86	66.36 ± 8.3	62.67 ± 8.98	69.26 ± 8.6	63.47 ± 8.59
	FPFS-3NN(P)	88.61 ± 2.51	65.99 ± 7.5	64.44 ± 8.14	69.99 ± 7.87	65.83 ± 7.54
	FPFS-3NN(S)	88.01 ± 2	64.35 ± 5.82	62.53 ± 6.52	69.23 ± 6.3	64.03 ± 6.01
	FPFS-3NN(K)	87.76 ± 2.2	63.65 ± 6.44	61.84 ± 7.16	68.6 ± 5.97	63.27 ± 6.6
	IFPIFSC	**91.39 ± 2.91**	**75.66 ± 9.25**	**73.18 ± 9.1**	**73.97 ± 8.64**	**74.16 ± 8.73**
Teaching Assistant Evaluation	Fuzzy 3NN	72.06 ± 5.53	59.99 ± 8.74	58.06 ± 8.36	57.23 ± 8.83	58.08 ± 8.3
	FSSC	63.6 ± 4.17	49.63 ± 13.36	45.98 ± 6.28	43.62 ± 6.25	45.41 ± 6.25
	FussCyier	63.69 ± 4.33	49.43 ± 12.15	46.09 ± 6.47	43.33 ± 6.56	45.53 ± 6.49
	HDFSSC	69.37 ± 4.66	55.55 ± 7.82	54.2 ± 7.07	53.37 ± 7.17	54.06 ± 6.99
	FPFSCC	69.12 ± 5.83	54.57 ± 9.52	53.77 ± 8.73	52.49 ± 9.16	53.68 ± 8.75
	FPFSNHC	60.86 ± 4.75	47.85 ± 14.61	41.84 ± 7.21	39.41 ± 6.38	41.3 ± 7.13
	FPFS-EC	75.53 ± 5.42	64.65 ± 9.06	63.2 ± 8.24	62.67 ± 8.51	63.29 ± 8.13
	FPFS-AC	**75.75 ± 4.67**	**64.96 ± 7.6**	**63.6 ± 6.96**	**62.9 ± 7.29**	**63.63 ± 7.01**
	FPFS-CMC	75.62 ± 4.75	64.92 ± 7.88	63.41 ± 7.08	62.7 ± 7.39	63.43 ± 7.12
	FPFS-3NN(P)	72.44 ± 5.48	59.41 ± 9.17	58.48 ± 8.34	57.54 ± 8.68	58.66 ± 8.22
	FPFS-3NN(S)	72.39 ± 5.07	58.98 ± 8.43	58.39 ± 7.7	57.5 ± 7.97	58.58 ± 7.61
	FPFS-3NN(K)	72.3 ± 5.19	58.86 ± 8.64	58.26 ± 7.88	57.37 ± 8.19	58.45 ± 7.79
	IFPIFSC	75.65 ± 4.48	64.43 ± 7.28	63.31 ± 6.78	62.6 ± 6.91	63.47 ± 6.72
Wine	Fuzzy 3NN	82.24 ± 4.86	73.79 ± 7.79	72.06 ± 7.39	72.22 ± 7.54	73.36 ± 7.3
	FSSC	96.26 ± 2.39	94.88 ± 3.1	95.3 ± 2.99	94.63 ± 3.46	94.38 ± 3.58
	FussCyier	96.44 ± 2.21	94.97 ± 3.1	95.42 ± 2.89	94.91 ± 3.19	94.66 ± 3.31
	HDFSSC	95.36 ± 2.66	93.49 ± 3.7	93.84 ± 3.61	93.35 ± 3.84	93.03 ± 3.99
	FPFSCC	92.43 ± 2.53	89.31 ± 3.6	89.99 ± 3.4	88.89 ± 3.79	88.65 ± 3.8
	FPFSNHC	95.54 ± 2.82	93.79 ± 3.74	94.41 ± 3.53	93.47 ± 4.23	93.31 ± 4.24
	FPFS-EC	97.64 ± 1.69	96.59 ± 2.42	97.04 ± 2.1	96.61 ± 2.45	96.46 ± 2.53
	FPFS-AC	95.87 ± 3.02	94.62 ± 3.45	94.82 ± 3.82	94.11 ± 4.42	93.81 ± 4.52
	FPFS-CMC	97.22 ± 2.64	96.15 ± 3.51	96.52 ± 3.31	96 ± 3.9	95.84 ± 3.96
	FPFS-3NN(P)	97.19 ± 2.15	96.03 ± 2.94	96.46 ± 2.72	95.93 ± 3.13	95.79 ± 3.22
	FPFS-3NN(S)	97.3 ± 2.28	96.25 ± 2.98	96.61 ± 2.87	96.13 ± 3.25	95.95 ± 3.42
	FPFS-3NN(K)	96.74 ± 2.54	95.59 ± 3.1	95.91 ± 3.2	95.34 ± 3.59	95.11 ± 3.8
	IFPIFSC	**98.24 ± 1.71**	**97.65 ± 2.12**	**97.79 ± 2.16**	**97.56 ± 2.36**	**97.36 ± 2.57**
Parkinsons[sic]	Fuzzy 3NN	85.38 ± 4.25	81.81 ± 6.39	78.34 ± 6.89	79.19 ± 6.29	85.38 ± 4.25
	FSSC	73.79 ± 6.35	72.76 ± 4.16	79.88 ± 5.09	71.49 ± 5.98	73.79 ± 6.35
	FussCyier	73.9 ± 6.44	73.25 ± 3.95	80.51 ± 4.79	71.73 ± 6.01	73.9 ± 6.44
	HDFSSC	78.21 ± 6.16	75.13 ± 5.07	82.04 ± 5.57	75.41 ± 6.11	78.21 ± 6.16
	FPFSCC	74.92 ± 6.14	68.07 ± 8.01	70.61 ± 10.07	68.22 ± 8.38	74.92 ± 6.14
	FPFSNHC	73.9 ± 6.51	72.86 ± 4.3	79.94 ± 5.16	71.58 ± 6.15	73.9 ± 6.51
	FPFS-EC	**95.85 ± 3.15**	**94.37 ± 4.71**	**95.15 ± 4.12**	**94.48 ± 4.17**	**95.85 ± 3.15**
	FPFS-AC	92.97 ± 4.27	91.04 ± 5.81	90.83 ± 6.1	90.56 ± 5.67	92.97 ± 4.27
	FPFS-CMC	95.03 ± 3.29	92.85 ± 4.62	94.67 ± 4.17	93.5 ± 4.24	95.03 ± 3.29
	FPFS-3NN(P)	94.41 ± 3.8	93.31 ± 5.26	92.03 ± 5.21	92.38 ± 5.01	94.41 ± 3.8
	FPFS-3NN(S)	93.95 ± 3.62	93.2 ± 5.11	90.83 ± 5.59	91.6 ± 5.03	93.95 ± 3.62
	FPFS-3NN(K)	93.95 ± 3.62	93.2 ± 5.11	90.83 ± 5.59	91.6 ± 5.03	93.95 ± 3.62
	IFPIFSC	95.23 ± 3.15	93.22 ± 4.51	94.99 ± 4.17	93.73 ± 4.11	95.23 ± 3.15

Table 2. *Cont.*

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Sonar	Fuzzy 3NN	82.5 ± 5.73	83.3 ± 5.77	82.04 ± 5.89	82.15 ± 5.89	82.5 ± 5.73
	FSSC	74.92 ± 7.5	75.5 ± 7.88	74.44 ± 7.62	74.42 ± 7.7	74.92 ± 7.5
	FussCyier	72.12 ± 5.63	73.68 ± 5.82	72.79 ± 5.66	71.94 ± 5.73	72.12 ± 5.63
	HDFSSC	69.38 ± 7.7	69.75 ± 7.96	69.46 ± 7.91	69.17 ± 7.82	69.38 ± 7.7
	FPFSCC	69.22 ± 6.77	69.38 ± 6.92	68.95 ± 6.84	68.82 ± 6.96	69.22 ± 6.77
	FPFSNHC	71.06 ± 5.46	72.63 ± 5.63	71.76 ± 5.44	70.87 ± 5.57	71.06 ± 5.46
	FPFS-EC	86.57 ± 4.79	87.37 ± 4.69	86.22 ± 4.88	86.34 ± 4.9	86.57 ± 4.79
	FPFS-AC	84.99 ± 5.18	86.2 ± 4.96	84.47 ± 5.38	84.62 ± 5.41	84.99 ± 5.18
	FPFS-CMC	85.53 ± 4.78	86.33 ± 4.73	85.22 ± 4.92	85.29 ± 4.92	85.53 ± 4.78
	FPFS-3NN(P)	86.77 ± 4.62	**88.1 ± 4.35**	86.21 ± 4.83	86.42 ± 4.87	86.77 ± 4.62
	FPFS-3NN(S)	86.19 ± 4.77	87.82 ± 4.48	85.56 ± 4.97	85.79 ± 5.04	86.19 ± 4.77
	FPFS-3NN(K)	86.19 ± 4.77	87.82 ± 4.48	85.56 ± 4.97	85.79 ± 5.04	86.19 ± 4.77
	IFPIFSC	**86.88 ± 5.15**	87.83 ± 5.35	**86.47 ± 5.25**	**86.65 ± 5.26**	**86.88 ± 5.15**
Seeds	Fuzzy 3NN	90.32 ± 3.44	87.35 ± 4.44	85.48 ± 5.16	85.36 ± 5.4	85.48 ± 5.16
	FSSC	94.1 ± 2.08	91.54 ± 2.96	91.14 ± 3.12	91.08 ± 3.18	91.14 ± 3.12
	FussCyier	94.13 ± 2.23	91.63 ± 3.14	91.19 ± 3.34	91.15 ± 3.37	91.19 ± 3.34
	HDFSSC	93.17 ± 2.13	90.34 ± 3.11	89.76 ± 3.2	89.76 ± 3.19	89.76 ± 3.2
	FPFSCC	90.48 ± 3.32	86.35 ± 4.91	85.71 ± 4.98	85.68 ± 5.02	85.71 ± 4.98
	FPFSNHC	93.52 ± 2.46	90.92 ± 3.43	90.29 ± 3.69	90.28 ± 3.71	90.29 ± 3.69
	FPFS-EC	93.14 ± 2.59	90.18 ± 3.98	89.71 ± 3.89	89.58 ± 4	89.71 ± 3.89
	FPFS-AC	93.49 ± 2.59	90.71 ± 3.9	90.24 ± 3.89	90.11 ± 3.95	90.24 ± 3.89
	FPFS-CMC	93.05 ± 2.74	90.02 ± 4.03	89.57 ± 4.11	89.45 ± 4.19	89.57 ± 4.11
	FPFS-3NN(P)	92.86 ± 2.38	89.82 ± 3.5	89.29 ± 3.58	89.23 ± 3.61	89.29 ± 3.58
	FPFS-3NN(S)	93.02 ± 2.66	90.06 ± 3.94	89.52 ± 4	89.46 ± 4.03	89.52 ± 4
	FPFS-3NN(K)	92.79 ± 2.51	89.77 ± 3.73	89.19 ± 3.76	89.14 ± 3.78	89.19 ± 3.76
	IFPIFSC	**95.49 ± 2.11**	**93.59 ± 3.07**	**93.24 ± 3.17**	**93.19 ± 3.25**	**93.24 ± 3.17**
Parkinson Acoustic	Fuzzy 3NN	75.96 ± 5.94	76.71 ± 5.98	75.96 ± 5.94	75.78 ± 6.01	75.96 ± 5.94
	FSSC	79.75 ± 5.69	80.34 ± 5.56	79.75 ± 5.69	79.63 ± 5.77	79.75 ± 5.69
	FussCyier	80 ± 5.79	80.5 ± 5.71	80 ± 5.79	79.9 ± 5.85	80 ± 5.79
	HDFSSC	**82.58 ± 4.79**	**83.03 ± 4.65**	**82.58 ± 4.79**	**82.51 ± 4.85**	**82.58 ± 4.79**
	FPFSCC	79.96 ± 5.08	80.73 ± 5.16	79.96 ± 5.08	79.83 ± 5.12	79.96 ± 5.08
	FPFSNHC	79.08 ± 5.57	79.63 ± 5.51	79.08 ± 5.57	78.97 ± 5.62	79.08 ± 5.57
	FPFS-EC	75.71 ± 7.05	76.05 ± 7.09	75.71 ± 7.05	75.62 ± 7.07	75.71 ± 7.05
	FPFS-AC	80.67 ± 5.63	81.23 ± 5.66	80.67 ± 5.63	80.58 ± 5.66	80.67 ± 5.63
	FPFS-CMC	75.79 ± 6.75	76.14 ± 6.89	75.79 ± 6.75	75.72 ± 6.76	75.79 ± 6.75
	FPFS-3NN(P)	80.38 ± 5.33	80.98 ± 5.28	80.38 ± 5.33	80.26 ± 5.4	80.38 ± 5.33
	FPFS-3NN(S)	79.79 ± 5.6	80.41 ± 5.51	79.79 ± 5.6	79.67 ± 5.69	79.79 ± 5.6
	FPFS-3NN(K)	80.46 ± 5.53	81.12 ± 5.47	80.46 ± 5.53	80.34 ± 5.61	80.46 ± 5.53
	IFPIFSC	82.54 ± 5.44	82.97 ± 5.39	82.54 ± 5.44	82.48 ± 5.48	82.54 ± 5.44
Ecoli	Fuzzy 3NN	92.08 ± 1.22	53.87 ± 3.94	60.13 ± 6.24	64.95 ± 5.85	68.34 ± 4.89
	FSSC	94.73 ± 1.31	70.9 ± 7.74	74.61 ± 4.46	81.39 ± 5.05	80.69 ± 4.41
	FussCyier	95.23 ± 1.19	73.87 ± 7.4	**75.16 ± 4.73**	**82.21 ± 5.03**	82.59 ± 4.08
	HDFSSC	94.99 ± 1.1	69.08 ± 6	74.43 ± 4.63	81.44 ± 4.4	81.41 ± 3.85
	FPFSCC	88.74 ± 1.78	47.56 ± 8.84	51.08 ± 8.31	56.28 ± 6.8	57.89 ± 5.7
	FPFSNHC	93.64 ± 1.39	64 ± 7.65	66.75 ± 7.76	74.49 ± 6.31	76.13 ± 4.98
	FPFS-EC	94.08 ± 1.28	68.97 ± 11.17	65.21 ± 8.02	74.07 ± 6.9	78.66 ± 4.75
	FPFS-AC	94.1 ± 1.12	72.12 ± 8.3	67.66 ± 6.71	74.88 ± 4.71	79.04 ± 4.06
	FPFS-CMC	93.94 ± 1.14	67.75 ± 9.7	64.38 ± 6.89	72.69 ± 5.24	78.18 ± 4.14
	FPFS-3NN(P)	94.49 ± 1.03	74.72 ± 8.65	65.59 ± 6.41	74.75 ± 5.57	81.31 ± 3.45
	FPFS-3NN(S)	95.18 ± 1.01	**78.06 ± 7.5**	70.1 ± 6.75	78.82 ± 5.3	83.66 ± 3.43
	FPFS-3NN(K)	**95.26 ± 1**	77.83 ± 7.43	70.88 ± 6.87	78.46 ± 5.68	**83.93 ± 3.34**
	IFPIFSC	94.8 ± 1.06	77.54 ± 7.7	71.43 ± 5.67	79.18 ± 4.76	81.73 ± 3.65

Table 2. *Cont.*

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Leaf	Fuzzy 3NN	96.14 ± 0.23	31.16 ± 4.69	31.18 ± 3.95	61.27 ± 4.03	31.94 ± 4.05
	FSSC	97.43 ± 0.34	66.6 ± 5.92	61.82 ± 5.22	70.9 ± 3.85	61.5 ± 5.13
	FussCyier	97.46 ± 0.35	66.76 ± 5.82	62.26 ± 5.23	71.58 ± 3.66	61.97 ± 5.21
	HDFSSC	97.6 ± 0.32	68.65 ± 5.49	64.47 ± 5.01	72.52 ± 3.51	63.97 ± 4.77
	FPFSCC	96.95 ± 0.32	59.05 ± 5.89	54.58 ± 5.07	67.86 ± 4.42	54.26 ± 4.75
	FPFSNHC	97.46 ± 0.3	66.45 ± 5.12	62.43 ± 4.6	72.58 ± 3.27	61.97 ± 4.52
	FPFS-EC	97.8 ± 0.3	71.26 ± 5.97	67.11 ± 5.04	74.37 ± 3.3	67.06 ± 4.54
	FPFS-AC	97.85 ± 0.28	72.46 ± 4.26	67.86 ± 4.56	74.59 ± 3.43	67.74 ± 4.27
	FPFS-CMC	97.74 ± 0.28	70.79 ± 4.49	66.38 ± 4.59	73.41 ± 3.59	66.15 ± 4.21
	FPFS-3NN(P)	97.78 ± 0.28	71.74 ± 4.52	66.47 ± 3.9	74.31 ± 4.11	66.65 ± 4.13
	FPFS-3NN(S)	97.94 ± 0.3	74.14 ± 4.72	68.83 ± 4.46	75.74 ± 4.15	69.12 ± 4.56
	FPFS-3NN(K)	97.92 ± 0.31	74.32 ± 4.83	68.6 ± 4.43	75.16 ± 4.04	68.82 ± 4.62
	IFPIFSC	**98.15 ± 0.26**	**76.88 ± 4.09**	**72.17 ± 3.95**	**76.88 ± 3.11**	**72.24 ± 3.87**
Ionosphere	Fuzzy 3NN	84.99 ± 3.61	89.17 ± 3.11	79.57 ± 4.86	81.66 ± 4.98	84.99 ± 3.61
	FSSC	64.1 ± 0.37	64.1 ± 0.37	50 ± 0	78.13 ± 0.27	64.1 ± 0.37
	FussCyier	64.1 ± 0.37	64.1 ± 0.37	50 ± 0	78.13 ± 0.27	64.1 ± 0.37
	HDFSSC	64.1 ± 0.37	64.1 ± 0.37	50 ± 0	78.13 ± 0.27	64.1 ± 0.37
	FPFSCC	84.88 ± 6.17	84.51 ± 6.72	83.52 ± 5.79	83.58 ± 6.36	84.88 ± 6.17
	FPFSNHC	82.6 ± 4.17	83.27 ± 5.08	78.43 ± 4.94	79.76 ± 5.1	82.6 ± 4.17
	FPFS-EC	89.55 ± 3.65	**91.98 ± 2.91**	85.94 ± 4.97	87.73 ± 4.71	89.55 ± 3.65
	FPFS-AC	88.81 ± 3.5	91.82 ± 2.63	84.79 ± 4.77	86.76 ± 4.52	88.81 ± 3.5
	FPFS-CMC	89.12 ± 2.91	91.59 ± 2.48	85.44 ± 3.98	87.28 ± 3.69	89.12 ± 2.91
	FPFS-3NN(P)	87.81 ± 2.84	91.11 ± 2.4	83.42 ± 3.83	85.51 ± 3.66	87.81 ± 2.84
	FPFS-3NN(S)	87.78 ± 3.11	90.9 ± 3.02	83.47 ± 4.01	85.53 ± 3.9	87.78 ± 3.11
	FPFS-3NN(K)	87.87 ± 3.09	91.03 ± 2.88	83.55 ± 4.04	85.62 ± 3.91	87.87 ± 3.09
	IFPIFSC	**91.14 ± 2.91**	91.26 ± 3.43	**89.54 ± 3.25**	**90.19 ± 3.22**	**91.14 ± 2.91**
Libras Movement	Fuzzy 3NN	95.9 ± 0.55	73.7 ± 3.83	69.23 ± 4.06	69.07 ± 4.07	69.22 ± 4.13
	FSSC	93.13 ± 0.75	54.48 ± 5.59	48.39 ± 5.68	52.25 ± 5.52	48.44 ± 5.62
	FussCyier	93.39 ± 0.72	55.52 ± 5.74	50.39 ± 5.58	53.84 ± 4.93	50.42 ± 5.43
	HDFSSC	93.94 ± 0.72	59.18 ± 5.98	54.49 ± 5.51	58.01 ± 4.74	54.58 ± 5.41
	FPFSCC	93.17 ± 0.75	53.71 ± 5.96	48.71 ± 5.7	52.09 ± 5.15	48.81 ± 5.66
	FPFSNHC	93.15 ± 0.8	53.32 ± 6.05	48.64 ± 6	53 ± 5.49	48.64 ± 5.99
	FPFS-EC	97.01 ± 0.56	80.44 ± 4.62	77.59 ± 4.18	77.63 ± 4.17	77.56 ± 4.2
	FPFS-AC	97.33 ± 0.52	82.59 ± 3.83	80.09 ± 3.78	79.78 ± 3.61	79.94 ± 3.87
	FPFS-CMC	96.95 ± 0.59	79.7 ± 4.51	77.27 ± 4.26	77.64 ± 4.35	77.14 ± 4.4
	FPFS-3NN(P)	96.85 ± 0.59	80.47 ± 4.13	76.42 ± 4.4	76.22 ± 4.21	76.39 ± 4.44
	FPFS-3NN(S)	96.74 ± 0.6	79.61 ± 3.79	75.55 ± 4.43	75.26 ± 4.24	75.56 ± 4.5
	FPFS-3NN(K)	96.75 ± 0.62	79.67 ± 3.96	75.62 ± 4.57	75.31 ± 4.37	75.61 ± 4.65
	IFPIFSC	**97.89 ± 0.46**	**86.55 ± 3.16**	**84.21 ± 3.53**	**83.65 ± 3.59**	**84.17 ± 3.43**
Dermatology	Fuzzy 3NN	91.22 ± 1.2	77.95 ± 3.66	71.9 ± 4.71	72.01 ± 4.45	73.66 ± 3.6
	FSSC	**99.15 ± 0.55**	**97.36 ± 1.75**	**97.14 ± 1.88**	**97.13 ± 1.86**	**97.46 ± 1.65**
	FussCyier	98.62 ± 0.81	95.82 ± 2.32	96.27 ± 2.11	95.78 ± 2.41	95.85 ± 2.44
	HDFSSC	98.87 ± 0.72	96.51 ± 2.2	96.5 ± 2.16	96.31 ± 2.28	96.61 ± 2.16
	FPFSCC	93.85 ± 1.33	83.13 ± 3.86	82.69 ± 3.73	81.68 ± 3.88	81.56 ± 3.99
	FPFSNHC	97.75 ± 0.96	93.65 ± 2.52	93.77 ± 2.72	93.08 ± 2.95	93.25 ± 2.88
	FPFS-EC	98.03 ± 0.77	94.21 ± 2.19	93.98 ± 2.4	93.69 ± 2.41	94.1 ± 2.31
	FPFS-AC	98.83 ± 0.78	96.53 ± 2.33	96.23 ± 2.5	96.23 ± 2.5	96.5 ± 2.33
	FPFS-CMC	97.66 ± 0.81	92.75 ± 2.6	92.65 ± 2.74	92.42 ± 2.66	92.98 ± 2.43
	FPFS-3NN(P)	97.4 ± 0.88	92.31 ± 2.57	91.98 ± 2.76	91.76 ± 2.8	92.21 ± 2.65
	FPFS-3NN(S)	98.31 ± 0.72	94.78 ± 2.3	94.65 ± 2.37	94.46 ± 2.38	94.94 ± 2.17
	FPFS-3NN(K)	98.24 ± 0.76	94.66 ± 2.3	94.5 ± 2.38	94.28 ± 2.44	94.72 ± 2.27
	IFPIFSC	99.01 ± 0.72	96.93 ± 2.37	96.72 ± 2.3	96.67 ± 2.41	97.02 ± 2.15

Table 2. *Cont.*

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Breast Cancer Wisconsin	Fuzzy 3NN	92.02 ± 2.1	91.97 ± 2.24	90.96 ± 2.39	91.36 ± 2.29	92.02 ± 2.1
	FSSC	93.64 ± 2.33	93.4 ± 2.49	93.03 ± 2.64	93.16 ± 2.52	93.64 ± 2.33
	FussCyier	93.53 ± 2.3	94.3 ± 2.18	91.98 ± 2.88	92.88 ± 2.58	93.53 ± 2.3
	HDFSSC	92.85 ± 2.27	93 ± 2.29	91.69 ± 2.79	92.22 ± 2.52	92.85 ± 2.27
	FPFSCC	93.34 ± 1.9	93.09 ± 2.09	92.73 ± 2.12	92.85 ± 2.04	93.34 ± 1.9
	FPFSNHC	93.81 ± 2.25	94.69 ± 2.11	92.22 ± 2.79	93.19 ± 2.52	93.81 ± 2.25
	FPFS-EC	95.27 ± 1.65	95.09 ± 1.94	94.88 ± 1.68	94.94 ± 1.75	95.27 ± 1.65
	FPFS-AC	95.08 ± 1.58	94.85 ± 1.79	94.76 ± 1.74	94.74 ± 1.68	95.08 ± 1.58
	FPFS-CMC	95.03 ± 1.74	94.84 ± 1.9	94.62 ± 1.94	94.67 ± 1.86	95.03 ± 1.74
	FPFS-3NN(P)	**96.63 ± 1.43**	**96.75 ± 1.6**	**96.07 ± 1.59**	**96.37 ± 1.54**	**96.63 ± 1.43**
	FPFS-3NN(S)	96.54 ± 1.52	96.68 ± 1.69	95.96 ± 1.68	96.27 ± 1.63	96.54 ± 1.52
	FPFS-3NN(K)	96.54 ± 1.52	96.68 ± 1.69	95.96 ± 1.68	96.27 ± 1.63	96.54 ± 1.52
	IFPIFSC	95.69 ± 1.43	95.57 ± 1.59	95.28 ± 1.6	95.38 ± 1.54	95.69 ± 1.43
HCV Data	Fuzzy 3NN	97.17 ± 0.53	54.58 ± 11.24	48.12 ± 12.36	67.13 ± 10.33	92.94 ± 1.31
	FSSC	97.29 ± 0.62	64.38 ± 8.68	63.6 ± 11.47	69.32 ± 7.91	93.23 ± 1.55
	FussCyier	97.32 ± 0.61	65.17 ± 9.47	62.55 ± 11.3	69.64 ± 8.84	93.31 ± 1.52
	HDFSSC	96.73 ± 0.96	62.71 ± 8.67	**64.74 ± 11.14**	67.65 ± 6.87	91.82 ± 2.41
	FPFSCC	95.95 ± 0.99	51.7 ± 13.03	50.43 ± 11.24	65.59 ± 10.15	89.88 ± 2.48
	FPFSNHC	97.15 ± 0.64	63.69 ± 12.44	54.98 ± 11	68.58 ± 6.68	92.87 ± 1.61
	FPFS-EC	97.11 ± 0.57	60.45 ± 14.64	47.08 ± 10	82.26 ± 10.98	92.78 ± 1.42
	FPFS-AC	**97.97 ± 0.58**	**73.93 ± 14.51**	55.96 ± 10.7	76.71 ± 10.02	**94.92 ± 1.45**
	FPFS-CMC	97.04 ± 0.55	63.74 ± 13.69	48.65 ± 10.22	76.46 ± 10.7	92.6 ± 1.38
	FPFS-3NN(P)	97 ± 0.33	56.97 ± 9.95	38.03 ± 5.65	**84.43 ± 9.4**	92.51 ± 0.84
	FPFS-3NN(S)	97.3 ± 0.41	67.66 ± 12.12	43.88 ± 7.76	80.49 ± 9.48	93.26 ± 1.04
	FPFS-3NN(K)	97.3 ± 0.41	67.22 ± 11.88	43.88 ± 7.76	80.4 ± 9.54	93.26 ± 1.04
	IFPIFSC	97.92 ± 0.52	70.56 ± 10.8	57.48 ± 12.03	74.69 ± 7.55	94.81 ± 1.29
Parkinson's Disease Classification	Fuzzy 3NN	71.27 ± 3.19	61.36 ± 4.28	60.41 ± 3.76	60.68 ± 3.93	71.27 ± 3.19
	FSSC	38.3 ± 7	47.68 ± 4.78	48 ± 4.87	37.76 ± 6.63	38.3 ± 7
	FussCyier	62.3 ± 16.08	47.44 ± 6.03	49.01 ± 2.09	44.4 ± 11.95	62.3 ± 16.08
	HDFSSC	62.52 ± 15.96	47.31 ± 6.73	49.01 ± 2.22	45.17 ± 13.02	62.52 ± 15.96
	FPFSCC	74.56 ± 3.9	69.04 ± 4.1	72.65 ± 4.7	69.79 ± 4.35	74.56 ± 3.9
	FPFSNHC	73.79 ± 2.84	67.85 ± 3.19	70.99 ± 4.09	68.52 ± 3.36	73.79 ± 2.84
	FPFS-EC	94.1 ± 2.37	92.32 ± 3.28	92.24 ± 3.28	92.22 ± 3.12	94.1 ± 2.37
	FPFS-AC	93.63 ± 1.88	91.87 ± 2.76	91.38 ± 2.66	91.55 ± 2.46	93.63 ± 1.88
	FPFS-CMC	90.9 ± 2.32	88.37 ± 3.44	87.72 ± 2.89	87.94 ± 2.94	90.9 ± 2.32
	FPFS-3NN(P)	92.39 ± 1.93	91.11 ± 2.59	88.57 ± 3.4	89.62 ± 2.79	92.39 ± 1.93
	FPFS-3NN(S)	91.67 ± 1.88	89.89 ± 2.54	87.84 ± 3.3	88.69 ± 2.73	91.67 ± 1.88
	FPFS-3NN(K)	91.67 ± 1.85	89.96 ± 2.44	87.74 ± 3.36	88.66 ± 2.73	91.67 ± 1.85
	IFPIFSC	**94.95 ± 1.56**	**93.76 ± 2.18**	**92.96 ± 2.56**	**93.27 ± 2.12**	**94.95 ± 1.56**
Mice Protein Expression	Fuzzy 3NN	99.89 ± 0.12	99.58 ± 0.43	99.56 ± 0.47	99.56 ± 0.46	99.55 ± 0.47
	FSSC	98.67 ± 0.48	95.01 ± 1.8	94.9 ± 1.86	94.83 ± 1.88	94.69 ± 1.92
	FussCyier	98.75 ± 0.48	95.33 ± 1.78	95.22 ± 1.85	95.14 ± 1.88	94.99 ± 1.9
	HDFSSC	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	FPFSCC	99.98 ± 0.05	99.91 ± 0.18	99.91 ± 0.19	99.91 ± 0.19	99.91 ± 0.19
	FPFSNHC	99.98 ± 0.05	99.93 ± 0.16	99.93 ± 0.16	99.92 ± 0.16	99.92 ± 0.18
	FPFS-EC	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	FPFS-AC	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	FPFS-CMC	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	FPFS-3NN(P)	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	FPFS-3NN(S)	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	FPFS-3NN(K)	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	IFPIFSC	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**

Table 2. *Cont.*

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Semeion Handwritten Digit	Fuzzy 3NN	97.23 ± 0.71	**97.67 ± 1.47**	86.67 ± 3.56	91.16 ± 2.66	97.23 ± 0.71
	FSSC	44.16 ± 2.96	57.54 ± 0.36	68.98 ± 1.66	40.62 ± 2.23	44.16 ± 2.96
	FussCyier	76.2 ± 2.65	64.06 ± 1.33	84.36 ± 2.19	64.53 ± 2.41	76.2 ± 2.65
	HDFSSC	89.45 ± 1.75	73.53 ± 2.52	88.22 ± 2.91	78 ± 2.7	89.45 ± 1.75
	FPFSCC	66.56 ± 7.62	60.04 ± 2.83	75.92 ± 4.98	56.13 ± 6.03	66.56 ± 7.62
	FPFSNHC	80.18 ± 2.34	65.7 ± 1.66	85.25 ± 2.8	67.85 ± 2.51	80.18 ± 2.34
	FPFS-EC	96.65 ± 0.9	92.33 ± 2.98	88.4 ± 3.78	90.11 ± 2.86	96.65 ± 0.9
	FPFS-AC	95.2 ± 1.37	88.24 ± 4.48	83.75 ± 4.72	85.68 ± 4.24	95.2 ± 1.37
	FPFS-CMC	94.46 ± 1.15	85.4 ± 3.78	82.85 ± 3.92	83.93 ± 3.42	94.46 ± 1.15
	FPFS-3NN(P)	96.62 ± 0.77	94.26 ± 2.45	86.1 ± 3.52	89.53 ± 2.62	96.62 ± 0.77
	FPFS-3NN(S)	96.62 ± 0.77	94.26 ± 2.45	86.1 ± 3.52	89.53 ± 2.62	96.62 ± 0.77
	FPFS-3NN(K)	96.62 ± 0.77	94.26 ± 2.45	86.1 ± 3.52	89.53 ± 2.62	96.62 ± 0.77
	IFPIFSC	**98.14 ± 0.75**	97.32 ± 1.85	**92.16 ± 3.47**	**94.42 ± 2.43**	**98.14 ± 0.75**
Car Evaluation	Fuzzy 3NN	94.43 ± 0.71	79.11 ± 2.84	62.39 ± 4.61	66.95 ± 4.89	88.86 ± 1.41
	FSSC	72.2 ± 1.04	38.09 ± 1.48	57.24 ± 3.43	34.39 ± 1.83	44.39 ± 2.08
	FussCyier	80.38 ± 1.08	44.49 ± 1.83	65.07 ± 3.52	45.43 ± 2.22	60.76 ± 2.16
	HDFSSC	86.66 ± 1.05	55.65 ± 2.45	76.71 ± 4.15	60.53 ± 3.05	73.32 ± 2.09
	FPFSCC	84.99 ± 1.43	58.65 ± 4.56	75.17 ± 4.82	62.41 ± 5.01	69.98 ± 2.87
	FPFSNHC	79.61 ± 1.06	42.66 ± 2.21	63.24 ± 3.35	43.42 ± 2.78	59.21 ± 2.11
	FPFS-EC	97.46 ± 0.54	90.01 ± 2.56	89.04 ± 3.82	89.25 ± 2.96	94.91 ± 1.07
	FPFS-AC	97.79 ± 0.56	90.51 ± 3.23	**92.62 ± 2.64**	**91.24 ± 2.86**	95.57 ± 1.13
	FPFS-CMC	97.42 ± 0.62	89.93 ± 3.01	88.59 ± 3.57	88.88 ± 2.95	94.85 ± 1.24
	FPFS-3NN(P)	97.7 ± 0.69	89.07 ± 3.8	90.77 ± 3.76	89.62 ± 3.7	95.41 ± 1.38
	FPFS-3NN(S)	97.77 ± 0.64	89.4 ± 3.55	91.12 ± 3.47	89.99 ± 3.39	95.54 ± 1.28
	FPFS-3NN(K)	97.75 ± 0.65	89.39 ± 3.61	91.03 ± 3.45	89.93 ± 3.42	95.49 ± 1.29
	IFPIFSC	**98.03 ± 0.42**	**91.27 ± 3.03**	90.41 ± 3.19	90.59 ± 2.64	**96.06 ± 0.85**
Wireless Indoor Localization	Fuzzy 3NN	99.13 ± 0.28	98.29 ± 0.55	98.26 ± 0.56	98.26 ± 0.56	98.26 ± 0.56
	FSSC	97.5 ± 0.42	95.42 ± 0.71	95 ± 0.83	94.99 ± 0.84	95 ± 0.83
	FussCyier	97.62 ± 0.4	95.64 ± 0.68	95.24 ± 0.8	95.24 ± 0.8	95.24 ± 0.8
	HDFSSC	96.73 ± 0.57	93.9 ± 1.04	93.46 ± 1.15	93.46 ± 1.15	93.46 ± 1.15
	FPFSCC	91.39 ± 0.88	83.12 ± 1.73	82.79 ± 1.76	82.61 ± 1.78	82.79 ± 1.76
	FPFSNHC	94.64 ± 0.75	89.79 ± 1.36	89.27 ± 1.5	89.33 ± 1.48	89.27 ± 1.5
	FPFS-EC	94.86 ± 0.79	89.83 ± 1.57	89.73 ± 1.58	89.73 ± 1.58	89.73 ± 1.58
	FPFS-AC	95.63 ± 0.59	91.4 ± 1.18	91.26 ± 1.19	91.26 ± 1.19	91.26 ± 1.19
	FPFS-CMC	94.54 ± 0.69	89.22 ± 1.37	89.09 ± 1.39	89.1 ± 1.38	89.09 ± 1.39
	FPFS-3NN(P)	95.27 ± 0.73	90.71 ± 1.43	90.54 ± 1.46	90.57 ± 1.45	90.54 ± 1.46
	FPFS-3NN(S)	95.05 ± 0.73	90.28 ± 1.41	90.11 ± 1.46	90.14 ± 1.44	90.11 ± 1.46
	FPFS-3NN(K)	96.32 ± 0.67	92.8 ± 1.29	92.64 ± 1.33	92.67 ± 1.32	92.64 ± 1.33
	IFPIFSC	**99.15 ± 0.24**	**98.32 ± 0.47**	**98.3 ± 0.48**	**98.3 ± 0.48**	**98.3 ± 0.48**
Mean Performance Results	Fuzzy 3NN	89.1 ± 2.42	75.91 ± 4.92	72.31 ± 5.51	76.27 ± 5.06	78.7 ± 3.99
	FSSC	82.93 ± 2.53	73.21 ± 4.9	73.38 ± 4.66	72.95 ± 4.25	73.58 ± 4.08
	FussCyier	86.01 ± 2.9	73.98 ± 4.86	74.51 ± 4.5	74.96 ± 4.49	77.12 ± 4.48
	HDFSSC	87.43 ± 2.91	75.51 ± 4.69	76.26 ± 4.69	77.25 ± 4.55	79.42 ± 4.44
	FPFSCC	86.25 ± 3.08	72.2 ± 5.91	73.07 ± 5.94	73.55 ± 5.58	75.43 ± 4.9
	FPFSNHC	87.2 ± 2.49	75.07 ± 5.28	75.64 ± 5.21	75.39 ± 4.4	77.92 ± 4.13
	FPFS-EC	93.17 ± 2.1	84.82 ± 5.04	82.56 ± 4.91	85.91 ± 4.43	86.92 ± 3.5
	FPFS-AC	93.2 ± 2.1	85.81 ± 4.87	83.25 ± 4.85	85.63 ± 4.35	87.36 ± 3.48
	FPFS-CMC	92.68 ± 2.1	84.03 ± 4.97	81.73 ± 4.9	84.63 ± 4.4	86.24 ± 3.55
	FPFS-3NN(P)	93.04 ± 1.95	84.75 ± 4.47	81.39 ± 4.46	85.38 ± 4.28	86.67 ± 3.31
	FPFS-3NN(S)	92.99 ± 1.95	85.45 ± 4.41	81.9 ± 4.53	85.38 ± 4.19	86.84 ± 3.26
	FPFS-3NN(K)	93.03 ± 1.96	85.51 ± 4.43	81.98 ± 4.58	85.38 ± 4.21	86.89 ± 3.3
	IFPIFSC	**94.45 ± 1.83**	**88.21 ± 4.21**	**86.11 ± 4.31**	**87.98 ± 3.68**	**89.62 ± 3.03**

Acc, Pre, Rec, MacF, and MicF results and their standard deviations (SD) are presented in percentage. The best performance results are shown in bold.

Table 2 manifests that IFPIFSC exactly classifies the dataset “Mice Protein Expression” just as HDFSSC, FPFS-EC, FPFS-AC, FPFS-CMC, FPFS-3NN(P), FPFS-3NN(S), and FPFS-3NN(K) do. Moreover, in compliance with all performance metrics, the performance results of IFPIFSC for “Ionosphere”, “Zoo”, “Car Evaluation”, “Semeion Handwritten Digit”, “Parkinson’s Disease Classification”, “Seeds”, “Parkinsons[sic]”, “Breast Cancer Wisconsin”, “Dermatology”, “Wine”, and “Wireless Indoor Localization” are over 89%, 89%, 90%, 92%, 92%, 93%, 93%, 95%, 96%, 97%, and 98%, respectively. In addition, IFPIFSC produces the best results in all performance metrics in “Breast Tissue”, “Wine”, “Sonar” (except for Pre value), “Seeds”, “Leaf”, “Ionosphere” (except for Pre value), “Libras Movement”, “Parkinson’s Disease Classification”, “Semeion Handwritten Digit” (except for Pre value), “Car Evaluation” (except for Rec and MacF values), and “Wireless Indoor Localization”. Although IFPIFSC does not produce the best results in all performance metrics in “Parkinsons[sic]”, “Parkinson Acoustic”, and “HCV Data”, it generates the closest results to the best ones for these datasets, except for the Pre value in “Parkinsons[sic]” and the Rec and MacF values in “HCV Data”. Consequently, the mean performance results in Table 2 indicate that IFPIFSC is a more efficient classifier than other classifiers on the considered datasets.

IFPIFSC achieves exceptional classification performance due to its utilizing HPS, EPS, MPS, HsPS, JPS, and CPS over *ifpifs*-matrices space and Pearson correlation coefficient-based feature weight. Moreover, Table 3 consists of ranking numbers of the best results, while Table 4 includes a pairwise comparison of the ranking results.

Table 3. Ranking numbers of the best results for all fuzzy-based classifiers.

Classifiers	Acc	Pre	Rec	MacF	MicF	Total Rank
Fuzzy 3NN	0/20	1/20	0/20	0/20	0/20	1/100
FSSC	1/20	1/20	1/20	1/20	1/20	5/100
FussCyier	0/20	0/20	1/20	1/20	0/20	2/100
HDFSSC	2/20	2/20	3/20	2/20	2/20	11/100
FPFSCC	0/20	0/20	0/20	0/20	0/20	0/100
FPFSNHC	0/20	0/20	0/20	0/20	0/20	0/100
FPFS-EC	3/20	4/20	2/20	3/20	3/20	15/100
FPFS-AC	3/20	3/20	3/20	3/20	3/20	15/100
FPFS-CMC	1/20	1/20	1/20	1/20	1/20	5/100
FPFS-3NN(P)	2/20	3/20	2/20	3/20	2/20	12/100
FPFS-3NN(S)	1/20	2/20	1/20	1/20	1/20	6/100
FPFS-3NN(K)	2/20	1/20	1/20	1/20	2/20	7/100
IFPIFSC	12/20	9/20	12/20	11/20	12/20	56/100

Table 4. Ranking numbers of the best results of IFPIFSC over the others.

Classifiers	Acc	Pre	Rec	MacF	MicF
IFPIFSC versus Fuzzy 3NN	20	19	20	20	20
IFPIFSC versus FSSC	19	19	17	18	19
IFPIFSC versus FussCyier	19	20	18	19	20
IFPIFSC versus HDFSSC	18	19	17	18	19
IFPIFSC versus FPFSCC	20	20	20	20	20
IFPIFSC versus FPFSNHC	20	20	20	20	20
IFPIFSC versus FPFS-EC	18	16	19	16	17
IFPIFSC versus FPFS-AC	18	17	18	17	18
IFPIFSC versus FPFS-CMC	19	17	19	19	19
IFPIFSC versus FPFS-3NN(P)	19	17	18	18	19
IFPIFSC versus FPFS-3NN(S)	18	18	18	18	18
IFPIFSC versus FPFS-3NN(K)	18	18	18	18	18

Afterward, Table 5 provides the average Acc, Pre, Rec, MacF, and MicF results of IFPIFSC, SVM, DT, BT, RF, and AdaBoost for the datasets. Table 5 shows that IFPIFSC exactly classifies the dataset "Mice Protein Expression" just as SVM, DT, and RF do. Furthermore, according to all performance metrics, the performance results of IFPIFSC for "Ionosphere", "Zoo", "Car Evaluation", "Semeion Handwritten Digit", "Parkinson's Disease Classification", "Parkinsons[sic]", "Seeds", "Breast Cancer Wisconsin", "Dermatology", "Wine", and "Wireless Indoor Localization" are over 89%, 89%, 90%, 92%, 92%, 92%, 93%, 95%, 96%, 96%, and 98%, respectively. In addition, IFPIFSC produces the best results in all performance metrics in "Breast Tissue", "Teaching Assistant Evaluation" (except for Pre value), "Parkinsons[sic]", "Sonar", "Seeds", "Parkinson Acoustic", "Libras Movement", "Parkinson's Disease Classification", and "Wireless Indoor Localization". Moreover, though IFPIFSC does not generate the best results in all performance metrics in "Wine", "Leaf", and "Dermatology", it produces the closest results to the best ones for these datasets. As a result, the mean performance results in Table 5 demonstrate that IFPIFSC is a more efficacious classifier than other classifiers on the considered datasets. Moreover, Table 6 consists of ranking numbers of the best results, while Table 7 includes a pairwise comparison of the ranking results.

Table 5. Simulation results of the non-fuzzy-based classifiers.

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Zoo	SVM	98.51 ± 1.14	92.64 ± 6.1	89.79 ± 7.5	94.88 ± 4.55	94.84 ± 3.99
	DT	96.97 ± 1.19	83.19 ± 8.82	76.02 ± 9.61	87.71 ± 4.33	89.6 ± 4.18
	BT	82.45 ± 1.28	40.57 ± 1.15	14.76 ± 0.96	57.71 ± 1.15	40.57 ± 1.15
	RF	**98.9 ± 1.02**	**94.96 ± 6.42**	**90.36 ± 9.41**	**96.58 ± 4**	**96.24 ± 3.54**
	AdaBoost	82.45 ± 1.28	40.57 ± 1.15	14.76 ± 0.96	57.71 ± 1.15	40.57 ± 1.15
	IFPIFSC	98.67 ± 1.03	93.29 ± 6.43	89.38 ± 8.15	95.44 ± 4.57	95.44 ± 3.59
Breast Tissue	SVM	89.03 ± 3.56	69.42 ± 11.53	66.12 ± 11.07	68.75 ± 9.86	67.1 ± 10.68
	DT	88.8 ± 2.98	69.31 ± 9.82	65.28 ± 9.38	69.12 ± 8.28	66.41 ± 8.94
	BT	89.75 ± 2.92	71.85 ± 9.62	68.49 ± 8.9	70.29 ± 7.01	69.26 ± 8.75
	RF	89.81 ± 3.16	70.99 ± 11.25	68.13 ± 9.58	72.02 ± 7.23	69.42 ± 9.48
	AdaBoost	89.75 ± 2.92	71.85 ± 9.62	68.49 ± 8.9	70.29 ± 7.01	69.26 ± 8.75
	IFPIFSC	**90.97 ± 2.22**	**74.85 ± 8.12**	**71.96 ± 7.36**	**73.15 ± 6.85**	**72.91 ± 6.67**
Teaching Assistant Evaluation	SVM	68.03 ± 5.17	53.94 ± 8.34	52.2 ± 7.79	51.88 ± 7.59	52.05 ± 7.76
	DT	69.76 ± 6.08	55.26 ± 9.74	54.57 ± 9.12	53.91 ± 9.36	54.65 ± 9.12
	BT	70.5 ± 5.73	56.88 ± 9.61	55.73 ± 8.63	55.36 ± 8.75	55.75 ± 8.59
	RF	74.56 ± 4.68	**62.81 ± 7.87**	61.73 ± 7.08	61.16 ± 7.32	61.85 ± 7.01
	AdaBoost	70.5 ± 5.73	56.88 ± 9.61	55.73 ± 8.63	55.36 ± 8.75	55.75 ± 8.59
	IFPIFSC	**74.62 ± 5**	62.75 ± 8.12	**61.79 ± 7.6**	**61.34 ± 7.85**	**61.94 ± 7.5**
Wine	SVM	96.78 ± 2.27	95.45 ± 3.06	95.43 ± 3.12	95.19 ± 3.33	95.16 ± 3.4
	DT	93.69 ± 3.48	91.08 ± 5.21	90.91 ± 4.92	90.59 ± 5.27	90.54 ± 5.22
	BT	61.17 ± 6.34	41.79 ± 9.64	35.54 ± 10.93	58.21 ± 6.24	41.76 ± 9.51
	RF	**98.68 ± 1.44**	**98.02 ± 2.18**	**98.29 ± 1.85**	**98.06 ± 2.11**	**98.03 ± 2.16**
	AdaBoost	61.17 ± 6.34	41.79 ± 9.64	35.54 ± 10.93	58.21 ± 6.24	41.76 ± 9.51
	IFPIFSC	97.98 ± 2.05	97.37 ± 2.39	97.45 ± 2.62	97.2 ± 2.88	96.97 ± 3.08
Parkinsons[sic]	SVM	86.67 ± 3.06	87.29 ± 6.23	75.98 ± 5.42	79.04 ± 5.42	86.67 ± 3.06
	DT	86.67 ± 5.38	82.75 ± 6.98	83.58 ± 6.64	82.57 ± 6.51	86.67 ± 5.38
	BT	89.23 ± 5.56	86.83 ± 7.62	84.09 ± 8.17	84.87 ± 7.85	89.23 ± 5.56
	RF	90.67 ± 3.76	90.23 ± 5.54	84.64 ± 6.44	86.45 ± 5.57	90.67 ± 3.76
	AdaBoost	88.87 ± 6.85	87.64 ± 8.09	81.6 ± 13.94	87.43 ± 5.42	88.87 ± 6.85
	IFPIFSC	**94.67 ± 3.97**	**92.72 ± 5.29**	**93.76 ± 5.59**	**92.95 ± 5.13**	**94.67 ± 3.97**
Sonar	SVM	76.2 ± 6.51	77.01 ± 6.77	75.69 ± 6.54	75.68 ± 6.7	76.2 ± 6.51
	DT	71.87 ± 6.85	72.33 ± 6.94	71.67 ± 6.83	71.51 ± 6.92	71.87 ± 6.85
	BT	85.04 ± 5.91	85.65 ± 6	84.75 ± 5.94	84.84 ± 5.97	85.04 ± 5.91
	RF	83.7 ± 6	84.73 ± 5.82	83.34 ± 6.1	83.37 ± 6.2	83.7 ± 6
	AdaBoost	84.37 ± 5.16	85.05 ± 5.18	83.99 ± 5.22	84.12 ± 5.26	84.37 ± 5.16
	IFPIFSC	**87.45 ± 5.13**	**88.26 ± 5.01**	**87.04 ± 5.23**	**87.21 ± 5.27**	**87.45 ± 5.13**

Table 5. *Cont.*

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Seeds	SVM	94.44 ± 2.41	92.12 ± 3.55	91.67 ± 3.61	91.58 ± 3.67	91.67 ± 3.61
	DT	94.29 ± 2.57	92.03 ± 3.69	91.43 ± 3.85	91.38 ± 3.91	91.43 ± 3.85
	BT	88.7 ± 14.81	83.41 ± 22.36	83.05 ± 22.21	85.64 ± 16.15	83.05 ± 22.21
	RF	95.27 ± 2.28	93.39 ± 3.25	92.9 ± 3.42	92.87 ± 3.44	92.9 ± 3.42
	AdaBoost	88.7 ± 14.81	83.41 ± 22.36	83.05 ± 22.21	85.64 ± 16.15	83.05 ± 22.21
	IFPIFSC	**95.68 ± 2.44**	**94.02 ± 3.39**	**93.52 ± 3.66**	**93.48 ± 3.71**	**93.52 ± 3.66**
Parkinson Acoustic	SVM	80.17 ± 6.12	80.85 ± 5.98	80.17 ± 6.12	80.03 ± 6.23	80.17 ± 6.12
	DT	72.54 ± 5.95	73.1 ± 6.17	72.54 ± 5.95	72.38 ± 5.98	72.54 ± 5.95
	BT	80.29 ± 5.46	81.03 ± 5.42	80.29 ± 5.46	80.16 ± 5.52	80.29 ± 5.46
	RF	80.46 ± 5.39	81.13 ± 5.48	80.46 ± 5.39	80.35 ± 5.43	80.46 ± 5.39
	AdaBoost	81.54 ± 5.76	82.21 ± 5.72	81.54 ± 5.76	81.43 ± 5.81	81.54 ± 5.76
	IFPIFSC	**81.88 ± 4.67**	**82.32 ± 4.62**	**81.88 ± 4.67**	**81.81 ± 4.72**	**81.88 ± 4.67**
Ecoli	SVM	93.91 ± 0.78	78.32 ± 9.3	51.95 ± 9.5	75.99 ± 6.39	79.29 ± 2.95
	DT	94.43 ± 1.16	71.31 ± 8.92	57.72 ± 7.98	75.75 ± 5.77	80.69 ± 4.34
	BT	95.28 ± 1.09	78.54 ± 9.7	67.64 ± 9.94	80.4 ± 5.47	83.49 ± 4.16
	RF	**95.8 ± 0.97**	**84.53 ± 5.55**	71.05 ± 9.06	**83.45 ± 4.34**	**85.69 ± 3.53**
	AdaBoost	95.28 ± 1.09	78.54 ± 9.7	67.64 ± 9.94	80.4 ± 5.47	83.49 ± 4.16
	IFPIFSC	94.85 ± 1.01	77.57 ± 7.86	**71.34 ± 6.66**	79.43 ± 4.82	81.82 ± 3.81
Leaf	SVM	96.96 ± 0.29	62.81 ± 5.15	53.46 ± 4.08	68.93 ± 4.32	54.47 ± 4.35
	DT	97.44 ± 0.36	66.56 ± 6.58	61.31 ± 5.54	70.92 ± 3.94	61.65 ± 5.4
	BT	97.84 ± 0.38	73.3 ± 6.15	67.39 ± 5.9	74.64 ± 4.25	67.62 ± 5.63
	RF	**98.4 ± 0.35**	**80.12 ± 5.24**	**75.43 ± 5.25**	**80.56 ± 3.94**	**75.94 ± 5.25**
	AdaBoost	97.84 ± 0.38	73.3 ± 6.15	67.39 ± 5.9	74.64 ± 4.25	67.62 ± 5.63
	IFPIFSC	98.11 ± 0.31	76.44 ± 4.84	71.4 ± 4.7	75.83 ± 3.95	71.59 ± 4.69
Ionosphere	SVM	87.18 ± 2.85	89.02 ± 2.87	83.44 ± 3.83	85.08 ± 3.61	87.18 ± 2.85
	DT	88.58 ± 3.32	87.84 ± 3.7	87.75 ± 3.69	87.61 ± 3.6	88.58 ± 3.32
	BT	**93.93 ± 2.7**	**94.57 ± 2.64**	**92.32 ± 3.41**	**93.21 ± 3.1**	**93.93 ± 2.7**
	RF	93.3 ± 2.7	93.55 ± 2.9	91.98 ± 3.24	92.58 ± 3.03	93.3 ± 2.7
	AdaBoost	93.25 ± 2.39	94.01 ± 2.43	91.43 ± 3.03	92.43 ± 2.75	93.25 ± 2.39
	IFPIFSC	91.43 ± 2.56	91.6 ± 2.69	89.87 ± 3.4	90.47 ± 2.95	91.43 ± 2.56
Libras Movement	SVM	95.86 ± 0.63	73.58 ± 4.58	68.99 ± 4.61	68.76 ± 4.83	68.97 ± 4.73
	DT	94.92 ± 0.88	65.79 ± 6.93	61.87 ± 6.68	63.13 ± 6.09	61.89 ± 6.62
	BT	96.09 ± 0.63	74.11 ± 4.32	70.63 ± 4.8	70.94 ± 5.08	70.64 ± 4.72
	RF	97.45 ± 0.57	83.09 ± 3.92	80.95 ± 4.21	80.78 ± 4.45	80.86 ± 4.29
	AdaBoost	96.09 ± 0.63	74.11 ± 4.32	70.63 ± 4.8	70.94 ± 5.08	70.64 ± 4.72
	IFPIFSC	**97.93 ± 0.5**	**86.88 ± 3.56**	**84.55 ± 3.78**	**83.9 ± 4.08**	**84.5 ± 3.77**
Dermatology	SVM	98.89 ± 0.55	96.57 ± 1.78	96.33 ± 1.86	96.25 ± 1.89	96.67 ± 1.66
	DT	98.12 ± 0.64	94.09 ± 2.55	93.37 ± 3.04	93.23 ± 2.67	94.35 ± 1.91
	BT	98.87 ± 0.58	96.08 ± 2.36	95.6 ± 2.94	95.53 ± 2.67	96.61 ± 1.75
	RF	**99.25 ± 0.57**	**97.79 ± 1.8**	**97.45 ± 1.94**	**97.51 ± 1.9**	**97.76 ± 1.71**
	AdaBoost	98.87 ± 0.58	96.08 ± 2.36	95.6 ± 2.94	95.53 ± 2.67	96.61 ± 1.75
	IFPIFSC	99.03 ± 0.58	96.83 ± 2.01	96.79 ± 1.84	96.67 ± 1.93	97.08 ± 1.75
Breast Cancer Wisconsin	SVM	95.29 ± 2.07	95.31 ± 2.36	94.67 ± 2.17	94.93 ± 2.21	95.29 ± 2.07
	DT	93.03 ± 2.37	92.56 ± 2.61	92.67 ± 2.5	92.56 ± 2.52	93.03 ± 2.37
	BT	96.64 ± 1.8	96.92 ± 1.8	95.96 ± 2.14	96.37 ± 1.95	96.64 ± 1.8
	RF	95.9 ± 1.76	95.88 ± 1.94	95.4 ± 1.94	95.6 ± 1.89	95.9 ± 1.76
	AdaBoost	**96.92 ± 1.65**	**97.12 ± 1.7**	**96.34 ± 1.92**	**96.68 ± 1.79**	**96.92 ± 1.65**
	IFPIFSC	95.57 ± 1.59	95.4 ± 1.82	95.18 ± 1.66	95.26 ± 1.69	95.57 ± 1.59
HCV Data	SVM	97.89 ± 0.7	70.03 ± 13.49	**62.44 ± 13.79**	72.6 ± 7.52	94.72 ± 1.75
	DT	97.18 ± 0.71	63.1 ± 11.34	53.11 ± 12.66	70.15 ± 8.79	92.95 ± 1.79
	BT	97.9 ± 0.52	**70.93 ± 12.7**	56.71 ± 11.99	75.35 ± 8.03	94.75 ± 1.29
	RF	97.76 ± 0.63	68.44 ± 14.52	54.28 ± 13.14	**76.44 ± 9.19**	94.41 ± 1.57
	AdaBoost	97.9 ± 0.52	**70.93 ± 12.7**	56.71 ± 11.99	75.35 ± 8.03	94.75 ± 1.29
	IFPIFSC	**97.92 ± 0.48**	70.4 ± 10.95	57.58 ± 11.82	72.78 ± 7.15	**94.8 ± 1.19**

Table 5. *Cont.*

Datasets	Classifiers	Acc ± SD	Pre ± SD	Rec ± SD	MacF ± SD	MicF ± SD
Parkinson's Disease Classification	SVM	74.6 ± 0.29	74.6 ± 0.29	50 ± 0	85.45 ± 0.19	74.6 ± 0.29
	DT	80.54 ± 3.35	74.46 ± 4.45	74.34 ± 4.9	74.25 ± 4.58	80.54 ± 3.35
	BT	91.28 ± 2.03	91.87 ± 2.78	84.75 ± 3.55	87.44 ± 3.09	91.28 ± 2.03
	RF	87.17 ± 2.22	87.78 ± 3.73	77.34 ± 3.92	80.53 ± 3.77	87.17 ± 2.22
	AdaBoost	90.29 ± 2.37	91 ± 3.08	82.89 ± 4.34	85.77 ± 3.84	90.29 ± 2.37
	IFPIFSC	**94.83 ± 1.85**	**93.6 ± 2.27**	**92.69 ± 3.05**	**93.08 ± 2.56**	**94.83 ± 1.85**
Mice Protein Expression	SVM	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	DT	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	BT	78.48 ± 0.01	13.93 ± 0.03	12.5 ± 0	24.45 ± 0.05	13.93 ± 0.03
	RF	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	AdaBoost	78.48 ± 0.01	13.93 ± 0.03	12.5 ± 0	24.45 ± 0.05	13.93 ± 0.03
	IFPIFSC	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
Semeion Handwritten Digit	SVM	97.81 ± 0.78	95.05 ± 2.51	92.57 ± 3.29	93.67 ± 2.36	97.81 ± 0.78
	DT	93.07 ± 1.53	81.28 ± 4.69	80.16 ± 3.67	80.48 ± 3.57	93.07 ± 1.53
	BT	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**	**100 ± 0**
	RF	96.7 ± 0.88	97.19 ± 1.53	84.12 ± 4.34	89.19 ± 3.31	96.7 ± 0.88
	AdaBoost	97.93 ± 1.19	96.35 ± 3.61	91.88 ± 4.03	93.89 ± 3.48	97.93 ± 1.19
	IFPIFSC	98.15 ± 0.73	97.21 ± 1.91	92.27 ± 3.25	94.49 ± 2.25	98.15 ± 0.73
Car Evaluation	SVM	92.53 ± 0.7	79.4 ± 4.27	75.88 ± 4.2	76.98 ± 3.69	85.07 ± 1.39
	DT	97.82 ± 0.48	90.42 ± 2.87	91.52 ± 3.65	90.66 ± 2.75	95.64 ± 0.96
	BT	98.51 ± 0.46	90.14 ± 3.25	94.03 ± 3.13	91.67 ± 3.06	97.02 ± 0.92
	RF	**98.94 ± 0.47**	**94.21 ± 2.84**	**96.29 ± 2.7**	**95.1 ± 2.61**	**97.88 ± 0.94**
	AdaBoost	98.51 ± 0.46	90.14 ± 3.25	94.03 ± 3.13	91.67 ± 3.06	97.02 ± 0.92
	IFPIFSC	97.97 ± 0.51	90.52 ± 3.27	90.47 ± 3.45	90.21 ± 2.58	95.94 ± 1.01
Wireless Indoor Localization	SVM	99 ± 0.35	98.02 ± 0.68	97.99 ± 0.69	97.99 ± 0.69	97.99 ± 0.69
	DT	98.52 ± 0.4	97.08 ± 0.78	97.05 ± 0.8	97.04 ± 0.8	97.05 ± 0.8
	BT	99.08 ± 0.31	98.18 ± 0.61	98.16 ± 0.63	98.16 ± 0.63	98.16 ± 0.63
	RF	99.09 ± 0.34	98.21 ± 0.66	98.19 ± 0.68	98.19 ± 0.68	98.19 ± 0.68
	AdaBoost	99.08 ± 0.31	98.18 ± 0.61	98.16 ± 0.63	98.16 ± 0.63	98.16 ± 0.63
	IFPIFSC	**99.15 ± 0.32**	**98.33 ± 0.64**	**98.31 ± 0.65**	**98.31 ± 0.65**	**98.31 ± 0.65**
Mean Performance Results	SVM	90.99 ± 2.01	83.07 ± 4.94	77.74 ± 4.96	82.68 ± 4.25	83.8 ± 3.43
	DT	90.41 ± 2.48	80.18 ± 5.64	77.84 ± 5.57	80.75 ± 4.78	83.16 ± 4.09
	BT	89.55 ± 2.92	76.33 ± 5.89	72.12 ± 5.98	78.26 ± 4.8	77.45 ± 4.64
	RF	93.59 ± 1.96	87.85 ± 4.62	84.12 ± 4.98	87.04 ± 4.02	88.85 ± 3.31
	AdaBoost	89.39 ± 3.02	76.16 ± 6.07	71.5 ± 6.46	78.01 ± 4.84	77.29 ± 4.74
	IFPIFSC	**94.34 ± 1.85**	**88.02 ± 4.26**	**85.86 ± 4.46**	**87.65 ± 3.78**	**89.44 ± 3.09**

Acc, Pre, Rec, MacF, and MicF results and their standard deviations (SD) are presented in percentage. The best performance results are shown in bold.

Table 6. Ranking numbers of the best results for all non-fuzzy-based classifiers.

Classifiers	Acc	Pre	Rec	MacF	MicF	Total Rank
SVM	1/20	1/20	2/20	1/20	1/20	6/100
DT	1/20	1/20	1/20	1/20	1/20	5/100
BT	2/20	3/20	2/20	2/20	2/20	11/100
RF	7/20	8/20	6/20	8/20	7/20	36/100
AdaBoost	1/20	2/20	1/20	1/20	1/20	6/100
IFPIFSC	11/20	9/20	11/20	10/20	11/20	52/100

Table 7. Ranking numbers of the best results of IFPIFSC over the others.

Classifiers	Acc	Pre	Rec	MacF	MicF
IFPIFSC versus SVM	20	19	17	20	20
IFPIFSC versus DT	20	20	19	19	20
IFPIFSC versus BT	15	15	16	14	15
IFPIFSC versus RF	12	11	13	11	12
IFPIFSC versus AdaBoost	16	16	17	15	16

5.4. Statistical Evaluation

The present subsection performs the Friedman test [32], a non-parametric test, and the Nemenyi test [33], a post hoc test, in a procedure proposed by Demšar [34] to analyze all performance results acquired in view of Acc, Pre, Rec, MacF, and MicF. The Friedman test generates a performance-based ranking of the classifiers for each dataset. Thus, the rank of 1 implies the best-performing classifier, the rank of 2 to the second best, etc. If the performances of the classifiers are equal, then it assigns the average of their possible ranks to their ranks. It then compares the average ranks and calculates χ^2_F, distributed with $k-1$ degree of freedom. Here, k denotes the classifiers' number. Afterward, a post hoc test, e.g., the Nemenyi test, is employed to determine the differences between the classifiers. The determined differences between any two classifiers more than critical distance are accepted as statistically significant.

This subsection calculates each classifier's average ranking using the Friedman test. Here, the number of fuzzy-based classifiers compared with IFPIFSC is 12, i.e., $k=13$, and the number of datasets is 20, i.e., $N=20$. Friedman test statistics of Acc, Pre, Rec, MacF, and MicF results, $\chi^2_F=108.60$, $\chi^2_F=106.69$, $\chi^2_F=90.48$, $\chi^2_F=108.51$, and $\chi^2_F=110.43$, respectively. For $k=13$ and $N=20$, the Friedman test critical value is 21.03 at the $\alpha=0.05$ significance level (for more details, see [40]). Since the Friedman test statistics of Acc (108.60), Pre (106.69), Rec (90.48), MacF (108.51), and MicF (110.43) are greater than the critical value 21.03; there is a significant difference between the performances of the classifiers. Hence, the null hypothesis "There are no performance differences between the classifiers" is rejected, and, thus, the Nemenyi test can be applied. For $k=13$, $N=20$, and $\alpha=0.05$, since the critical value for the infinite degrees of freedom in the table Studentized Range q is $cv=4.286$, the critical distance is $cd=\frac{cv}{\sqrt{2}}\times\sqrt{\frac{k\times(k+1)}{6\times N}}=\frac{4.286}{\sqrt{2}}\times\sqrt{\frac{8\times(8+1)}{6\times 20}}\approx 2.348$ according to the Nemenyi test. The critical diagrams produced by the Nemenyi test for the five performance metrics are presented in Figure 2. Figure 2 manifests that the performance differences between the average rankings of IFPIFSC and those of FPFS-CMC, FPFCC, Fuzzy kNN, FPFS-NHC, FSSC, FussCyier, and HDFSSC, are greater than the critical distance (4.0798). Figure 2 shows that even though the difference between the average rankings of IFPIFSC and FPFS-EC, FPFS-AC, FPFS-3NN(P), FPFS-3NN(S), and FPFS-3NN(K) is less than the critical distance (4.0798), IFPIFSC performs better than them concerning all performance metrics. Therefore, IFPIFSC outperforms the others statistically for all five performance metrics.

Secondly, this subsection calculates each classifier's average ranking using the Friedman test. Here, the number of non-fuzzy-based classifiers compared with IFPIFSC is 5, i.e., $k=6$, and the number of datasets is 20, i.e., $N=20$. Friedman test statistics of Acc, Pre, Rec, MacF, and MicF results, $\chi^2_F=48.65$, $\chi^2_F=45.28$, $\chi^2_F=39.93$, $\chi^2_F=45.64$, and $\chi^2_F=48.65$, respectively. For $k=6$ and $N=20$, the Friedman test critical value is 11.07 at the $\alpha=0.05$ significance level (for more details, see [40]). Since the Friedman test statistics of Acc (48.65), Pre (45.28), Rec (39.93), MacF (45.64), and MicF (48.65) are greater than the critical value 11.07; there is a significant difference between the performances of the classifiers. Thereby, the null hypothesis "There are no performance differences between the classifiers" is rejected, and, thus, the Nemenyi test can be applied. For $k=6$, $N=20$, and $\alpha=0.05$, since the value for the infinite degrees of freedom in the table Studentized Range q is 4.030, the critical distance is $\frac{4.030}{\sqrt{2}}\times\sqrt{\frac{6\times(6+1)}{6\times 20}}\approx 1.686$ according to the Nemenyi test.

The critical diagrams generated by the Nemenyi test for the five performance metrics are presented in Figure 3.

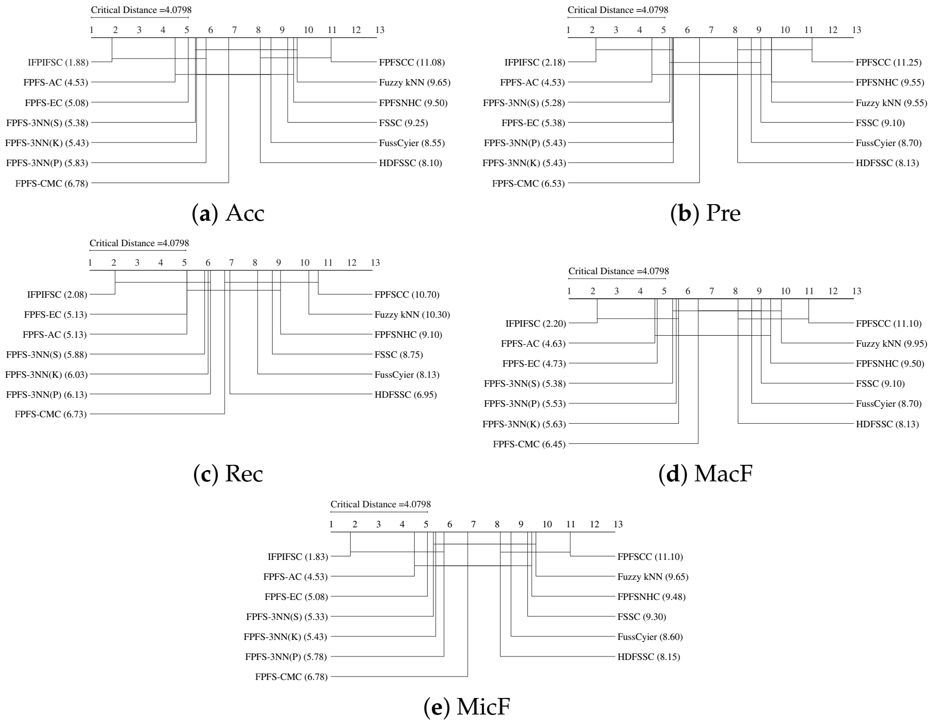

Figure 2. The critical diagrams obtained by the Friedman test and Nemenyi test at 0.05 significance level for the five performance criteria (for fuzzy-based classifiers).

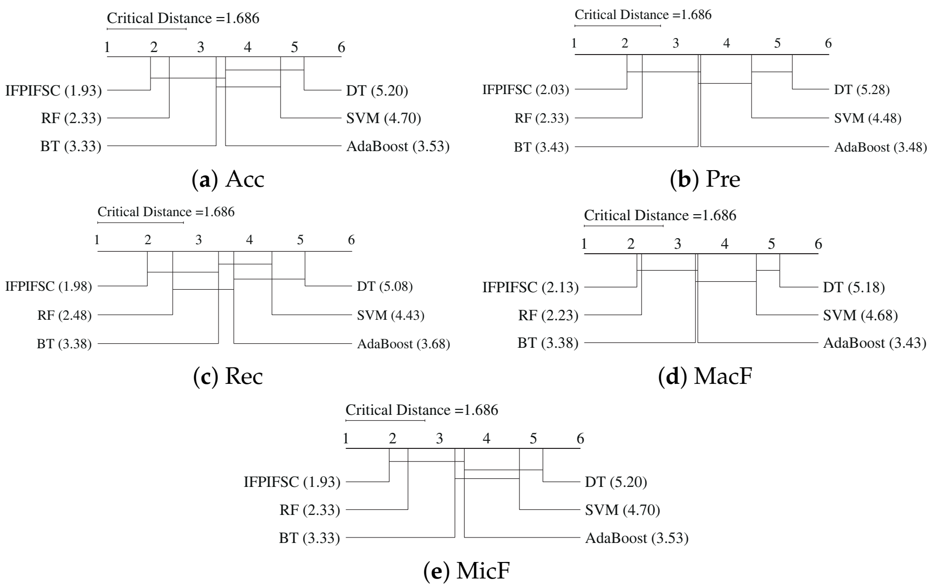

Figure 3. The critical diagrams obtained by the Friedman test and Nemenyi test at 0.05 significance level for the five performance criteria (for non-fuzzy-based classifiers).

Figure 3 demonstrates that the performance differences between the average rankings of IFPIFSC and those of AdaBoost (MacF), SVM, and DT are greater than the critical distance (1.686). In addition, Figure 3 indicates that IFPIFSC realizes better than RF, BT, and AdaBoost in terms of all performance metrics, although the difference between the average rankings of IFPIFSC, RF, BT, and AdaBoost is less than the critical distance (1.686). Therefore, IFPIFSC outperforms the others statistically for all five performance metrics.

5.5. Comparison of the Time Complexity

The present section compares the time complexities of the classifiers by employing a big O notation. From the pseudocode of IFPIFSC, the time complexity is $O(mn)$ since mn is higher than $m6$ for each test sample. Here, m and n are the numbers of the training samples and of their attributes, respectively. The time complexities, big O notation herein, of the compared classifiers are presented in Table 8.

Table 8. Time complexities based on big O notation of the classifiers.

Classifier	Time Complexity
Fuzzy *k*NN	$O(n^2 \log k)$
FSSC	$O(ml)$
FussCyier	$O(ml)$
HDFSSC	$O(ml)$
FPFSCC	$O(ml)$
FPFSNHC	$O(ml)$
FPFS-EC	$O(mn)$
FPFS-AC	$O(mn)$
FPFS-CMC	$O(m^2 + mn)$
FPFS-*k*NN(P)	$O(m^2 l)$
FPFS-*k*NN(S)	$O(m^2 l)$
FPFS-*k*NN(K)	$O(m^2 l)$
SVM	$O(m^2 n^2)$
DT	$O(mn \log n)$
BT	$O(tmn \log n)$
RF	$O(tmn \log n)$
AdaBoost	$O(tmn \log n)$
IFPIFSC	$O(mn)$

k is the number of the nearest neighbours, m is the sample number of the training data, n is the parameter number of the training data, l is the class number of the data, and t is the number of tree.

6. Conclusions

This study defined the concepts metrics, quasi-, semi-, and pseudo-metrics and similarities, quasi-, semi-, and pseudo-similarities over *ifpifs*-matrices. Thus, it theoretically contributed to the literature. Next, this study suggested five pseudo-metrics and seven pseudo-similarities over *ifpifs*-matrices. Hence, it confirmed the existence of the aforementioned contribution. Later, this study propounded IFPIFSC simultaneously using six of the proposed pseudo-similarities and applied it to a data classification problem. In this way, this study clarified how to construct *ifpifs*-matrices and apply them to real problems in data classification. Furthermore, it compared IFPIFSC with the well-known and state-of-the-art classifiers Fuzzy *k*NN, FSSC, FussCyier, HDFSSC, FPFSCC, FPFSNHC, FPFS-EC, FPFS-AC, FPFS-CMC, FPFS-*k*NN(P), FPFS-*k*NN(S), FPFS-*k*NN(K), SVM, DT, BT, RF, and AdaBoost and statistically analyzed the comparison results. Thereby, the present study manifested that the proposed method has the best performance results and, thus, is a pretty convenient method in supervised learning.

The success of the available classifiers has natural limits depending on datasets. Therefore, IFPIFSC has been designed to cope with these drawbacks. This classifier allows using novel multiple-similarity functions and threshold values. By this means, IFPIFSC is open to improvement: one of the ways to improve this proposed classifier is to define or use different similarity measures over *ifpifs*-matrices. The second is to adapt the values λ_1 and λ_2 used in the intuitionistic fuzzification of real data. The third is to use SDM methods constructed by *fpfs*- or *ifpifs*-matrices to use multiple pseudo-similarities similar to FPFS-AC and FPFS-CMC [12,13]. The fourth, to reduce the negative effects of the inconsistent data in the used datasets on the classification success, is to develop an effective preprocessing step that eliminates or excludes inconsistent data from the evaluation using rough sets [41,42]. The fifth is to develop similar classifiers constructed by interval-valued intuitionistic fuzzy parameterized interval-valued intuitionistic fuzzy soft matrices [43] modeling further uncertainties than intuitionistic fuzzy uncertainties. To struggle with different uncertainties, the sixth is to consider the new concepts, such as picture fuzzy sets [44,45], Pythagorean fuzzy sets [46,47], Fermatean fuzzy sets [48], q-rung orthopair fuzzy sets [49,50], T-spherical fuzzy sets [51,52], interval-valued fuzzy sets [53,54], interval-valued intuitionistic fuzzy sets [55], and bipolar fuzzy sets [56–58]. Finally, IFPIFSC can be customized to produce nearly 100% performance, especially for medical diagnosis problems. Classifiers whose codes are not shared privately or on online platforms, such as MathWorks and GitHub, are not included in the scope of this study.

Author Contributions: Ç.C. directed the project and supervised this study's findings. B.A. and T.A. devised the main conceptual ideas and developed the theoretical framework. S.M. and B.A. performed the experiment and statistical analyzes. S.M. wrote the manuscript with support from B.A., T.A. and S.E. S.E. reviewed and edited the paper. All the authors discussed the results and contributed to the final paper. All authors have read and agreed to the published version of the manuscript

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Abbreviations

fpfs-set [7]	Fuzzy Parameterized Fuzzy Soft Set
fpfs-matrix [8]	Fuzzy Parameterized Fuzzy Soft Matrix
ifs-set [17]	Intuitionistic Fuzzy Soft Set
ifps-set [18]	Intuitionistic Fuzzy Parameterized Soft Set
ifpfs-set [19]	Intuitionistic Fuzzy Parameterized Fuzzy Soft Set
ifpifs-set [20]	Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Set
ifpifs-matrix [21]	Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Matrix
SDM	Soft Decision-Making
*k*NN [59,60]	*k*-Nearest Neighbor
Fuzzy *k*NN [23]	Fuzzy *k*-Nearest Neigbors
FSSC [24]	Fuzzy Soft Set Classifier
FussCyier [25]	Fuzzy Soft Set Classifier Using Distance-Based Similarity Measure
HDFSSC [26]	Hamming Distance-Based Fuzzy Soft Set Classifier
FPFSCC [10]	Fuzzy Parameterized Fuzzy Soft Chebyshev Classifier
FPFSNHC [9]	Fuzzy Parameterized Fuzzy Soft Normalized Hamming Classifier
FPFS-EC [11]	Fuzzy Parameterized Fuzzy Soft Euclidean Classifier
FPFS-CMC [12]	Comparison Matrix-Based Fuzzy Parameterized Fuzzy Soft Classifier
FPFS-AC [13]	Fuzzy Parameterized Fuzzy Soft Aggregation Classifier
FPFS-*k*NN [14]	Fuzzy Parameterized Fuzzy Soft *k*-Nearest Neighbor
SVM [27]	Support Vector Machines
DT [28]	Decision Trees
BT [29]	Boosting Trees
AdaBoost [31]	Adaptive Boosting
RF [30]	Random Forests

IFPIFSC (In this paper)	Intuitionistic Fuzzy Parameterized Intuitionistic Fuzzy Soft Classifier
UCI-MLR [22]	UC Irvine Machine Learning Repository
Acc	Accuracy
Pre	Precision
Rec	Recall
MacF	Macro F-score
MicF	Micro F-score

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