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# LRS Bianchi-I universe model with domain wall in $f(R, T)$ gravity

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**Keywords:** LRS Bianchi-I universe, $f(R, T)$ gravitation theory, strange quark matter, domain wall

## Abstract

In this work, strange quark matter (SQM) and normal matter (NM) attached to domain wall (DW) matter distributions have been examined for locally rotationally symmetric (LRS) Bianchi-I space-time has been investigated. The model has been constructed in the framework of $f(R, T)$ theory for $f(R, T) = R + 2\mu T$ which is suggested as an alternative gravitation theory instead of Einstein's General Relativity Theory. Obtained modified field equations have been solved by using scale factors equation $A = B^m$ come from the proportion of expansion scalar $\theta$ to shear scalar $\varpi^2$. SQM and NM are added to DW with the help of equations of state (EoS) $p_m = \frac{1}{3}(\rho_m - 4B_c)$ and $p_m = (\gamma - 1)\rho_m$. Pressure and energy densities of all matter distributions have been obtained depending on cosmic time $t$ and their effects decrease by the time. It is found that DW matter behaves like stiff matter due to attaining DW pressure equal to DW energy density $p^{DW} = \rho^{DW}$. Also obtained values of scale factors and some kinematic quantities give an expanding universe for the model. Energy conditions and kinematic quantities of the model have been examined in the last section. In addition, General Relativity solutions of the model have been attained by assuming $\mu = 0$ in $f(R, T) = R + 2\mu T$. All solutions have been concluded in detail.

## 1. Introduction

The evolution of the Universe has always been one of the most important topics for researchers. For many years, Einstein field equations (EFE) have been used to study and explain this evolution. Einstein propounded the static universe model by attaching cosmological constant to EFE. However, this addition was inadequate in terms of some cosmological problems like flatness, the horizon, and the violation of energy conservation. Scientists have focused on developing Einstein's theory to overcome these problems and suggested some gravitational theories like vector-tensor, scalar-tensor, bimetric, and scalar field theories [1]. In the last decades, observations of Supernova Ia [2, 3], Cosmic Microwave Background Radiation (CMBR) [4, 5], and Wilkinson Microwave Anisotropy Probe (WMAP) [6] have revealed expansion of the Universe at an accelerating rate. In particular, it is insufficient to explain expanding universe phenomena with the theory, because of some contradictions such as red-shift and CMBR. For this reason, scientists have turned to investigation of dark energy representation such as k-essence, quintessence, and Tachyon or developments of alternative theories to Einstein's General Relativity Theory (GR). Lyra Theory [7], Brans-Dicke Theory (BD) [8], Barber Theories [9], $f(R, T)$ Gravitation Theory [10], etc. are given as alternative theory. Especially, besides Harko's theory, $f(R)$, $f(T)$, $f(G)$, $f(G, T)$, etc theories have been the center of attention to explain expanding universe behaviors with cosmic matter distributions. Here $R$, $T$, and $G$ symbolize the scalar curvature, the energy-momentum tensor trace and the Gauss Bonnet scalar respectively [11] have gotten solutions of higher dimensional Kaluza-Klein universe model filled with the perfect fluid matter in framework $f(R, T)$ gravitation theory [12] have studied Kaluza-Klein space-time in $f(R, T)$ theory with cosmological constant $\Lambda$ for perfect fluid matter distribution. Also, anisotropic and axially symmetric universe for perfect fluid matter has been examined in the framework of $f(R, T)$ gravity by [13, 14] have get solutions of the modified Einstein field equations (MEFEs) for $f(R, T)$ theory Kantowski-Sachs universe filled bulk viscous [15] have studied $f(R, T)$ gravity in Gödel space-time for perfect fluid matter.

LRS Bianchi-I space-time is a simple generalization of the homogenous-isotropic flat Robertson-Walker (RW) Universe and defines anisotropy on the system with different scale factors in each spatial direction. The LRS Bianchi type-I universe is important to examine the possible effect of the anisotropic universe on observations of late-time isotropic universe [16] have investigated anisotropic and homogenous Bianchi type-I space-time with massive string matter in GR theory [17] have examined the LRS Bianchi-I universe with imperfect fluid for time variables G and $\Lambda$ [18] has gotten solutions of Einstein Field equations (EFEs) for anisotropic Bianchi type-I model with magnetic field in string cosmology. Bianchi V universe model for bulk viscous fluid with cosmic string has been examined in $f(R, T)$ theory by [19, 20] has investigated Bianchi type-I universe in the framework of $f(R, T)$ gravity [21] have obtained solutions of MEFEs in $f(R, T)$ alternative theory by using quadratic equations of states (EoS). Scalar field solutions of Bianchi-I and Marder universe models have been solved in $f(R, T)$ gravity by Kabak and Aygün [22]. The Big Bang Theory predicts that the Universe cooled rapidly from initially high temperatures. As a result of the phase transitions that occurred due to the rapid cooling of the Universe, matter forms called topological defects which are interrupted or uninterrupted symmetry spontaneously formed at ultra-high temperatures [23]. These defects named cosmic strings, textures, monopoles, and domain walls are alterations of static fields in the context of quantum field theories [24]. Domain walls are the fields across from one minimum to another one and their structural behavior is associated with the density and the surface tension $\sigma_\omega$. Therefore, it is important from a cosmological perspective to examine the gravitational effect caused by topological defects. Because of these features, domain walls are studied in detail to analyze the dynamic structure of the Universe in the early stage. It is thought that some cosmic objects such as neutron stars and quark-gluon plasma may have originated from quark matter that occurred at the early universe phase when the ultra-high cosmic temperature is about 200 MeV [25–28]. This is called the quark-hadron transition and when quarks are considered massless and non-interactive, quark matter pressure $p_q$ is written as follows

$$p_q = \frac{\rho_q}{3} \tag{1}$$

Here $\rho_q$ symbolizes quark matter energy density [29]. The total pressure of the matter is calculated from the following equation.

$$\rho = \rho_q + B_c \tag{2}$$

$B_c$ given in the equation above is named the bag constant of quark matter [30, 31]. And total pressure of the matter is given as the equation below [32]

$$p = p_q - B_c \tag{3}$$

In addition, these equations, EoS for strange quark matter is given as the following form

$$p_m = \frac{1}{3}(\rho_m - 4B_c) \tag{4}$$

Here $p_m$ and $\rho_m$ are respectively symbols of particle pressure and density [33]. Quark-gluon plasma in perfect liquid structure has been brought into being by researchers at Brookhaven National Laboratory and EoS for quark-gluon plasma come from perfect fluid form is written as follows

$$p_m = (\gamma - 1)\rho_m \tag{5}$$

Here $\gamma$ is a constant and has values as $1 \leqslant \gamma \leqslant 2$ [33, 34] have solved Einstein field equations of n-dimensional SQM with the string cloud (SC) and electromagnetic field. Bianchi type-III universe with SQM attached to SC has been examined in GR theory by [35]. Also, [36] have investigated the Kaluza-Klein type universe for SQM and DW GR theory with cosmological constant $\Lambda$. Quark and SQM coupled with domain wall have been examined for Einstein-Rosen cylindrical symmetric universe in GR theory by [37, 38] have investigated SQM with SC and domain wall in plane symmetric space-time for Rosen's bimetric theory [29, 39] have studied SQM in Brans-Dicke theory for several anisotropic universe models [40] have attained solutions of MEFEs in $f(R, T)$ gravitation theory in the context of spherical symmetric kink universe for quark matter attached to SQM and domain walls [41] have solved field equations of Brans-Dicke and GR theories for SQM coupled with SC in Bianchi type-$VI_0$ space-time [42] have studied SQM for hypersurface homogeneous cosmological models in Lyra theory [43] have researched the n-dim FRW universe model with quark and SQM in the framework of Creation Field Cosmology [44] have gotten exact solutions of Bianchi-V universe model with magnetized quark matter in modified $f(R, T)$ theory of gravitation. Higher dimensional FRW universe model has been investigated in CFC theory for SQM attached to string cloud by Çağlar and Ayün [45, 46] have solved field equations of $f(R)$ theory in the presence of magnetized SQM for generalized anisotropic universe models. Bianchi type-II, VIII, and IX line elements in the context of $f(R, T)$ gravitation theory have been examined with domain walls by [47]. Maurya *et al* [48] have investigated $f(R, T)$ gravity for Bianchi V space-time with DW and quark matter. LRS Bianchi-I line element with bulk viscous and SC have been studied in Brans-Dicke theory by Çağlar [49].

Singh *et al* [50] have analyzed Bianchi type-I universe in $f(R, T)$ theory for SQM. Çağlar *et al* [51] have examined Ruban universe model filled quark and SQM coupling with DW in $f(R, T)$ gravity. Tangphati *et al* [52] have investigated $f(R, L_m, T)$ theory by using $f(R, L_m, T) = R + \alpha TL_m$ for on the internal structure of compact stars. Jorás [53] has studied on $f(R)$ theory in frame work phase transitions. Also Tanghati *et al* [54] have studied the conventional spherically symmetric metric with a rainbow metric for strange quark matter in form of anisotropic fluid. Júnior *et al* [55] have get solutions of field equations for Friedmann-Lemâitre-Robertson-Walker in $f(R, L_m, T)$ theory. A model for anisotropic interacting quark stars have been investigated for equations of states with pressure anisotropy by Tanghati *et al* [56]. Sheikh *et al* [57] have studied in $f(R, T)$ gravity for the gravitational collapse of a quark binding string fluid. Mangut *et al* [58] have examined $F(R, G)$ theory coupled with Born-Infeld for thermodynamics and naked singularity structure of the topological solutions.

In the light of the importance as mentioned above, in this study anisotropic and homogenous LRS Bianchi-I space-time filled with SQM and NM attached to DW have been investigated in $f(R, T)$ gravity. The paper is organized as; $f(R, T)$ gravity modified theory, obtaining the MEFEs and kinematic quantities is given in the second section and then solutions of MEFEs with SQM and NM are given in subsections of the third section, finally discussions and conclusions have given last section in detail.

## 2. Modified field equations of the model

In this work, Harko's theory has been examined for the LRS Bianchi-I universe. The field equations of $f(R, T)$ gravitation theory are written as [10, 59]

$$G_{ij} = [8\pi + 2f_T(T)]T_{ij} + [2pf_T(T) + f(T)]g_{ij} \tag{6}$$

Here subscript $T$ symbolizes differentiation. Also, Harkot *et al* [10] have suggested three models for $f(R, T)$ function of the model. In this study, the model of Harko *et al* [10] written as $f(R, T) = R + 2f(T)$ have been assumed as the function of theory. Here $f(T)$ can be written as $f(T) = \mu T$, hereby $f(R, T)$ function of rewritten as follow

$$f(R, T) = R + 2\mu T \tag{7}$$

Where $\mu$ is an arbitrary constant [60, 61]. Also energy momentum tensor of domain wall (DW) matter in the form of perfect fluid is given as

$$T_{ij}^{DW} = (\rho + p)u_i u_j - pg_{ij} \tag{8}$$

Here $\rho$ and $p$ respectively symbolize energy density and pressure of the DW matter distributions. On the other hand $u_i = (1, 0, 0, 0)$ is the 4-velocity in co-moving coordinates [62]. By using equations (7) and (8) in equation (6), MEFEs of Harko's theory rewritten as follow

$$G_{ij} = [8\pi + 2\mu]T_{ij} + [\mu\rho - \mu p]g_{ij} \tag{9}$$

It is clear to see that when one takes $\mu = 0$, $f(R, T)$ theory reduces to GR theory. The line element of axially symmetric LRS Bianchi-I universe model is written as follow [63]

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2) \tag{10}$$

$A$ and $B$ given in the equation above are scale factors of the LRS Bianchi-I space-time. By using equations (8)–(10), MEFEs of $f(R, T)$ gravitation theory for LRS Bianchi-I space time model with domain wall matter distributions have been attained as follows

$$2\frac{B_{tt}}{B} + \frac{B_t^2}{B^2} = \rho\mu - 8p\pi - 3p\mu \tag{11}$$

$$\frac{A_{tt}}{A} + \frac{B_{tt}}{B} + \frac{A_t B_t}{AB} = \rho\mu - 8p\pi - 3p\mu \tag{12}$$

$$\frac{B_t^2}{B^2} + 2\frac{A_t B_t}{AB} = 3\rho\mu + 8\rho\pi - p\mu \tag{13}$$

Here and after subscript $t$ is derivative with the cosmic time. Obtained three MEFEs equations (11)–(13) have four unknown parameters and one assumption such as kinematical quantities is needed to solve these equations. Some important kinematical quantities such as average scale factor $a$, spatial volume $V$, and Hubble parameter $H$ for LRS Bianchi-I are given as following equations.

$$a = (AB^2)^{\frac{1}{3}} \tag{14}$$

$$V = a^3 = AB^2 \tag{15}$$

$$H = \frac{V_t}{V} = \frac{A_t}{3A} + \frac{2B_t}{3B} \tag{16}$$

Also anisotropy parameter $AP$, expansion scalar $\theta$, and shear scalar $\varpi^2$ are defined and obtained as follows

$$AP = \frac{1}{3}\sum\left(\frac{H_i - H}{H}\right)^2 = \frac{2(B_t A - A_t B)^2}{(2B_t A + A_t B)^2} \tag{17}$$

$$\theta = u^i_{;i} = \frac{A_t}{A} + \frac{2B_t}{B} \tag{18}$$

$$\varpi^2 = \frac{1}{2}\varpi_{ij}\varpi^{ij} = \frac{1}{3}\frac{(B_t A - A_t B)^2}{A^2 B^2} \tag{19}$$

here $\varpi_{ij}$ is a tensor and given as follow

$$\varpi_{ij} = \frac{1}{2}(u_{i;k}h^k_j + u_{j;k}h^k_i) - \frac{1}{3}\theta h_{ij} \tag{20}$$

Also $h_{ij}$ projection tensor is qualified as $h_{ij} = g_{ij} - u_i u_j$. And finally, the deceleration parameter (DP) is defined by following equation [20].

$$q = -\frac{a_{tt}a}{a_t^2} = \frac{2(B_t A - A_t B)^2 - 3A_{tt}AB^2 - 6B_{tt}A^2 B}{(2B_t A + A_t B)^2} \tag{21}$$

The value of DP describes the flatness of the Universe. The positive and negative values of DP represent decelerating and inflation universe models respectively [64].

## 3. Solutions of modified field equations of the model

When it is used that proportion of expansion scalar $\theta$ to shear scalar $\varpi$ like studies of Mahanta [65] and Jokweni *et al* [66], the equation of the relationship between scale factors is obtained as

$$A = B^m \tag{22}$$

Here $m$ is an arbitrary constant, and observations of the redshift-velocity relation beyond galactic sources indicate that the Hubble expansion of the Universe is isotropic within an approximate range of 30%. More specifically, redshift analyses have determined that the ratio of shear $\varpi$ to the Hubble constant $H$ satisfies the constraint $\varpi/H \leqslant 0.3$ in the region near our galaxy. It is emphasized that for spatially homogeneous metrics, the condition of homogeneous expansion is met when $\varpi/\theta$ remains constant by Collin *et al* [67]. This implies a direct proportionality between the expansion scalar and the shear scalar, establishing a relationship between the metric potentials like in equation (22). By using equation (22) in equations (11)–(13), the scale factor $B$, domain wall energy density $\rho^{DW}$ and pressure $p^{DW}$ are attained as follows

$$B = [(m + 2)(k_1 t + k_2)]^{\frac{1}{m+2}} \tag{23}$$

$$p^{DW} = \frac{1}{2}\frac{(2m + 1)k_1^2}{(4\pi + \mu)(m + 2)^2(k_1 t + k_2)^2} \tag{24}$$

$$\rho^{DW} = \frac{1}{2}\frac{(2m + 1)k_1^2}{(4\pi + \mu)(m + 2)^2(k_1 t + k_2)^2} \tag{25}$$

Also, scale factor $A$ is obtained from equation (22) and equation (23) as follow

$$A = [(m + 2)(k_1 t + k_2)]^{\frac{m}{m+2}} \tag{26}$$

And $f(R, T)$ function given in equation (7) is attained as

$$f(R, T) = \frac{8(2m + 1)\pi k_1^2}{(4\pi + \mu)(m + 2)^2(k_1 t + k_2)^2} \tag{27}$$

In this study, SQMand normal matter have been attached to DW matter in $f(R, T)$ gravitation theory by using equations (4) and (5) with relations between domain wall tension $\sigma_\omega$ and matter quantities which are given in the following equations

$$p = p_m - \sigma_\omega \tag{28}$$

$$\rho = \rho_m + \sigma_\omega \tag{29}$$

Here $p_m$ and $\rho_m$ symbolize the matter pressure and energy density, respectively [24]. In subsections, solutions of SQM and NM with DW matter are given by using related equations.

### 3.1. Solutions for strange quark matter with DW model $p_m = \frac{1}{3}(\rho_m - 4B_c)$

It is possible to attach strange quark matter to domain wall by assuming DW pressure as $p^{DW} = p_m - \sigma_\omega$ and DW density as $\rho^{DW} = \rho_m + \sigma_\omega$ with equation (4). Hereby one can get SQM pressure, energy density coupled with DW and DW density as follows, respectively

$$p_m^{SQM} = p_q - B_c = \frac{1}{4}\frac{(2m+1)k_1^2}{(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} - B_c \tag{30}$$

$$\rho_m^{SQM} = \rho_q + B_c = \frac{3}{4}\frac{(2m+1)k_1^2}{(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} + B_c \tag{31}$$

$$\sigma_{\omega_m} = -\frac{1}{4}\frac{(2m+1)k_1^2}{(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} - B_c \tag{32}$$

Also quark matter pressure and energy density are calculated from equations (30) and (31) as

$$p_q = \frac{1}{4}\frac{(2m+1)k_1^2}{(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} \tag{33}$$

$$\rho_q = \frac{3}{4}\frac{(2m+1)k_1^2}{(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} \tag{34}$$

### 3.2. Solutions for normal matter with DW model $p_m = (\gamma - 1)\rho_m$

It is possible to add normal matter to domain wall by assuming DW pressure as $p^{DW} = p_m - \sigma_\omega$ and DW density as $\rho^{DW} = \rho_m + \sigma_\omega$ with equation (5). Hereby one can get NM pressure, energy density coupled with DW and DW density as follows, respectively

$$p_m^{NM} = \frac{(\gamma-1)(2m+1)k_1^2}{\gamma(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} \tag{35}$$

$$\rho_m^{NM} = \frac{(2m+1)k_1^2}{\gamma(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} \tag{36}$$

$$\sigma_\omega^{NM} = \frac{1}{2}\frac{(\gamma-2)(2m+1)k_1^2}{\gamma(4\pi+\mu)(m+2)^2(k_1t+k_2)^2} \tag{37}$$

## 4. Discussion and conclusion

In this study, LRS Bianchi type-I universe for strange quark matter and normal matter have been investigated by coupling with domain wall matter in $f(R, T) = R + 2f(T)$ model of $f(R, T)$ theory. Also, it is assumed that $f(T) = \mu T$ in the function of $f(R, T)$ model. By using the relation between scale factors as $A = B^m$, coming from the proportion of expansion scalar $\theta$ to shear scalar $\varpi^2$, solutions of MEFEs for the constructed model attained. The new line element can be written by using equations (23) and (26) in equation (10) as follow

$$ds^2 = dt^2 - [(m+2)(k_1t+k_2)]^{\frac{2m}{m+2}}dx^2 - [(m+2)(k_1t+k_2)]^{\frac{2}{m+2}}(dy^2+dz^2) \tag{38}$$

It is seen in equations (23), (26) and (38), that $m$ is an important constant and, must be $m \neq -2$ to attain meaningful and time depending line element. When $t = 0$, it gives the initial epoch of the Universe and the Universe model ends at $t = -k_2/k_1$. Constructed model solutions give an expanding universe with the time $t$ and it is shown for scale factor $A$ on figure 1.

Zel'dovich [68] proposed the possibility of a primordial stiff matter era while investigating the cosmological implications of an equation of state where the speed of sound equals the speed of light [69]. It is assumed with $p = \rho$ that the early universe made from cold baryon gas. The case of domain wall matter for constructed model in this work can also be compared to the stiff matter of the early universe that Zel'dovich referred to. Thus, one can say that DW matter behaves like stiff matter [70], because of domain wall pressure and energy density have been attained as $p^{DW} = \rho^{DW}$ seen in equations (24) and (25). DW matter effect decreases by the time and it is shown in figure 2. Besides, SQM and NM with DW effects decrease too by the time and it is given in figure 3.

It is known that $\gamma$ given in equation (5) has value $1 \leqslant \gamma \leqslant 2$ [33]. When $\gamma = 1$ assumed in equation (35), the value of normal matter pressure with DW becomes zero ($p_m^{NM} = 0$) and one can say that NM with DW matter distribution behaves like dust matter in condition $\gamma = 1$ [71, 72]. It is seen in equation (32) for $\mu > -4\pi$ that SQM domain wall tension $\sigma_\omega^{SQM}$ gets a negative value. Also, when $\mu > -4\pi$ assumed in equation (37), NM domain wall tension $\sigma_\omega^{NM}$ has a negative value due to $1 \leqslant \gamma \leqslant 2$. It means that SQM and NM with DW matter

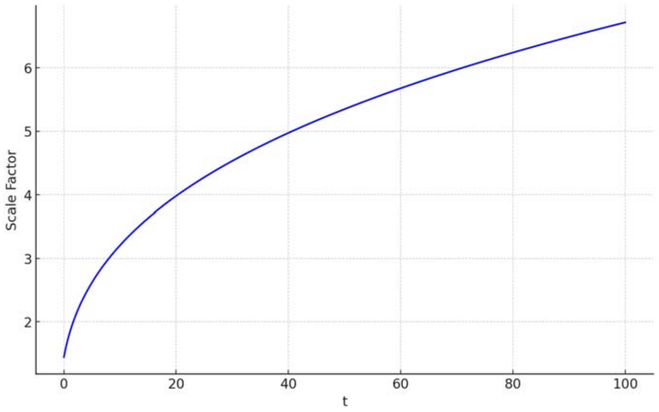

**Figure 1.** Variation of scale factor A against time for $k_1 = k_2 = m = 1$.

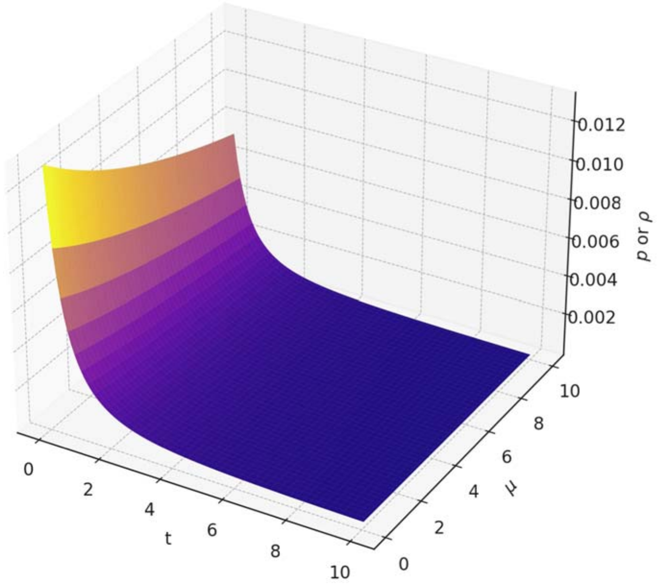

**Figure 2.** Variation of domain wall effect for $k_1 = k_2 = m = 1$.

behave like invisible matter in these situations for the constructed $f(R, T)$ model and agree with studies of Khadekar *et al* [36], Çağlar [51],Yılmaz [73] and Katore and Hatkar [74].

In this paper, energy conditions of constructed models such as Null Energy Condition (*NEC*), Weak Energy Condition (*WEC*),Dominant Energy Condition (*DEC*), and Strong Energy Condition (*SEC*) have been investigated. When $NEC = \rho^{eff} + p^{eff} \geqslant 0$, the NEC is provided for the model. Providing of *WEC* is attained in conditions of provided *NEC* and $\rho^{eff} \geqslant 0$, providing of *DEC* is attained in conditions of provided *NEC* and $\rho^{eff} - p^{eff} \geqslant 0$, also providing of *SEC* is attained in conditions of provided *NEC* and $\rho^{eff} + 3p^{eff} \geqslant 0$ [75–77]. In this work domain wall matter distribution has been assumed as matter filled the Universe model and evaluated effective values. *DEC* of DW matter is calculated as zero due to $p^{eff} = \rho^{eff}$. All energy conditions are provided for $m > -1/2$, $\mu \geqslant -4\pi$ and $t \neq -k_2/k_1$ and shown in figure 4. Also when it is assumed SQM and NM coupled DW, all energy conditions are provided and shown in figures 5 and 6.

Some kinematic quantities such as spatial volume $V$, Hubble parameter $H$, anisotropy parameter $AP$, expansion scalar $\theta$, and shear scalar $\varpi$ given in equations (15)–(19) have been calculated as follows

$$V = [(m+2)(k_1 t + k_2)]^{\frac{3m+2}{3(m+2)}} \tag{39}$$

$$H = \frac{k_1}{3(k_1 t + k_2)} \tag{40}$$

$$AP = \frac{2(m-1)^2}{(m+2)^2} \tag{41}$$

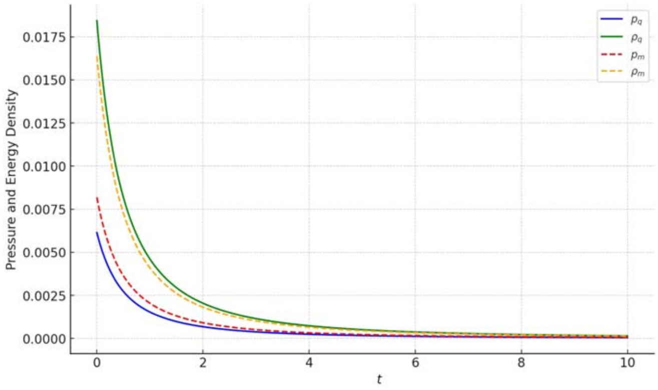

**Figure 3.** Variation of strange quark matter and normal matter with domain wall effect for $k_1 = k_2 = m = \mu = 1$ and $\gamma = 1.5$.

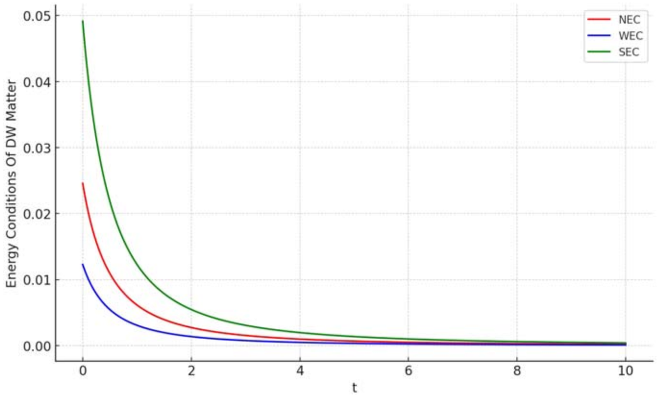

**Figure 4.** Variation of DW energy conditions against time for $k_1 = k_2 = m = \mu = 1$.

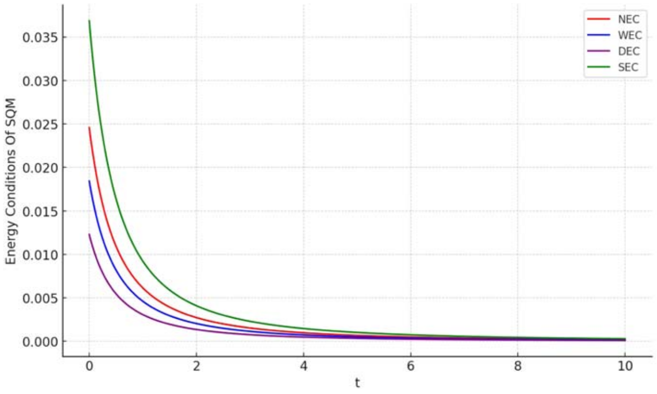

**Figure 5.** Variation of SQM with DW energy conditions against time for $k_1 = k_2 = m = \mu = 1$ and $\gamma = 1.5$.

$$\theta = \frac{k_1}{k_1 t + k_2} \tag{42}$$

$$\varpi^2 = \frac{1}{3}\left[\frac{(m-1)k_1}{(m+2)(k_1 t + k_2)}\right]^2 \tag{43}$$

$$q = 2 \tag{44}$$

Kinematic quantities, determined by the constant parameters $m$ and $k_1$, provide some distinctive cosmological interpretations for the constructed model. When $m = -2$, value of spatial volume $V$ gets zero and it is point to

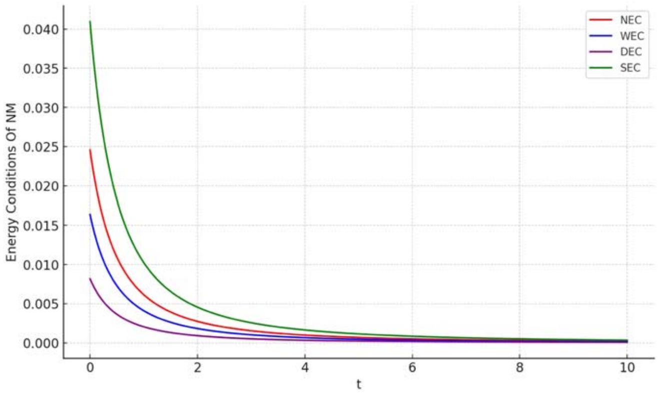

**Figure 6.** Variation of NM with DW energy conditions against time for $k_1 = k_2 = m = \mu = 1$ and $\gamma = 1.5$.

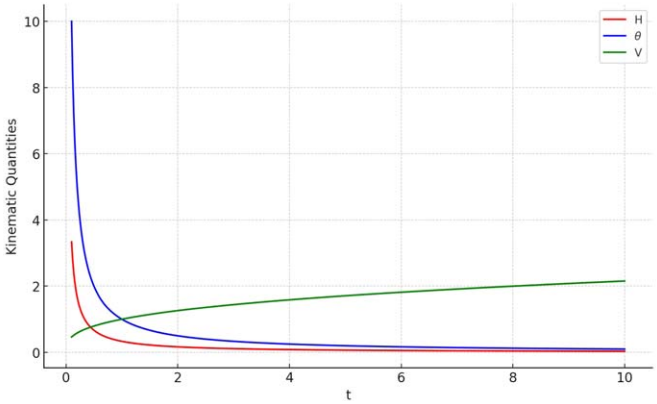

**Figure 7.** Variation of Hubble parameter, volume and expansion scalar against time for $k_1 = m = \mu = 1$ and $k_2 = 0$.

the initial moment of the Universe. If constant $k$ is assumed to be zero, static universe model has been attained. It is also crucial to analyze the anisotropy parameter given by equation (41) and to evaluate the relationship between the scale factors obtained in equation (22) for the constructed model. The arbitrary constant $m$ is directly proportional to anisotropy, while for $m = 1$, a homogeneous and isotropic FRW universe model is obtained. Additionally, infinite isotropy is achieved when $m = -2$. On the other hand, deceleration parameter has an important role in describing the expansion of the Universe. The expansion of the Universe is called according to the value of deceleration parameter such as exponential expansion for $q = -1$, accelerating power law expansion for $-1 \leqslant q \leqslant 0$, constant expansion for $q = 0$, and decelerating expansion for $q > 0$ [78]. In this work, it can be said that the constructed model has decelerating expansion due to attained $q = 2$ seen in equation (44). Attained solutions of kinematic quantities give that the constructed universe is expanding by the time $t$ and its shown in figure 7.

When one takes $\mu = 0$ in $f(R, T) = R + 2\mu T$, all attained solutions for $f(R, T)$ theory reduce to GR theory solutions of the constructed model. Thereby, domain wall pressure and energy density have been calculated from equations (24) and (25).

$$p_{GR}^{DW} = \rho_{GR}^{DW} = \frac{1}{8}\frac{(2m+1)k_1^2}{\pi(m+2)^2(k_1 t + k_2)^2} \tag{45}$$

SQM pressure and energy density with DW have been obtained in GR theory as follows

$$p_m^{SQMGR} = p_q - B_c = \frac{1}{16}\frac{(2m+1)k_1^2}{\pi(m+2)^2(k_1 t + k_2)^2} - B_c \tag{46}$$

$$\rho_m^{SQMGR} = \rho_q + B_c = \frac{3}{16}\frac{(2m+1)k_1^2}{\pi(m+2)^2(k_1 t + k_2)^2} + B_c \tag{47}$$

Also, DW tension with SQM has attained as

$$\sigma_{\omega}^{SQMGR} = -\frac{1}{16}\frac{(2m+1)k_1^2}{\pi(m+2)^2(k_1t+k_2)^2} - B_c \tag{48}$$

Quark matter pressure and energy density are calculated for GR theory from equations (46) and (47) as

$$p_q^{GR} = \frac{1}{16}\frac{(2m+1)k_1^2}{\pi(m+2)^2(k_1t+k_2)^2} \tag{49}$$

$$\rho_q^{GR} = \frac{3}{16}\frac{(2m+1)k_1^2}{\pi(m+2)^2(k_1t+k_2)^2} \tag{50}$$

NM pressure and energy density with DW have been obtained in GR theory as follows

$$p_m^{NMGR} = \frac{(\gamma-1)(2m+1)k_1^2}{4\gamma\pi(m+2)^2(k_1t+k_2)^2} \tag{51}$$

$$\rho_m^{NMGR} = \frac{(2m+1)k_1^2}{4\gamma\pi(m+2)^2(k_1t+k_2)^2} \tag{52}$$

Also, DW tension with NM has been attained in GR theory as

$$\sigma_{\omega}^{NMGR} = \frac{1}{8}\frac{(\gamma-2)(2m+1)k_1^2}{\gamma\pi(m+2)^2(k_1t+k_2)^2} \tag{53}$$

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## Conflict of interest

The author declare no competing interests.

## Data availability statement

No new data were created or analysed in this study.

## Declarations

## Ethics approval

The author approvals the ethicality of the article.

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## References

[1] Bekenstein J D 2004 Relativistic gravitation theory for the modified newtonian dynamics paradigm *Phys. Rev.* D **70** 083509
[2] Bahcall N A, Ostriker J P, Perlmutter S and Steinhardt P J 1999 The cosmic triangle: revealing the state of the universe *Sci.* **284** 1481–8
[3] Perlmutter S *et al* 1999 Measurements of $\omega$ and $\lambda$ from 42 high-redshift supernovae *Astrophys. J.* **517** 565
[4] Gawiser E and Silk J 2000 The cosmic microwave background radiation *Phys. Rep.* **333** 245–67

[5] Knop R A *et al* 2003 New constraints on $\omega$m, $\omega\lambda$ and w from an independent set of 11 high-redshift supernovae observed with the hubble space telescope *Astrophys. J.* **598** 102

[6] Spergel D N *et al* 2003 First-year wilkinson microwave anisotropy probe (wmap)$^*$ observations: determination of cosmological parameters *Astrophys. J. Suppl. Ser.* **148** 175

[7] Lyra G 1951 Uber eine modifikation der riemannschen geometrie *Math. Zeitschr.* **54** 52–64

[8] Brans C and Dicke R H 1961 Mach's principle and a relativistic theory of gravitation *Phys. Rev.* **124** 925

[9] Barber G A 1982 On two self-creation cosmologies *Gen. Relativ. Gravit.* **14** 117–36

[10] Harko T, Lobo F S, Nojiri S and Odintsov S D 2011 f (r, t) gravity *Phys. Rev.* D **84** 024020

[11] Reddy D, Naidu R and Satyanarayana B 2012 Kaluza-klein cosmological model in f (r, t) gravity *Int. J. Theor. Phys.* **51** 3222–7

[12] Sahoo P, Mishra B and Tripathy S 2016 Kaluza-klein cosmological model in f (r, t) gravity with $\lambda$ (t) *Indian J. Phys.* **90** 485–93

[13] Sahoo P, Mishra B and Chakradhar Reddy G 2014 Axially symmetric cosmological model in f (r, t) gravity *Eur. Phys. J. Plus* **129** 1–8

[14] Reddy D, Anitha S and Umadevi S 2014 Kantowski-sachs bulk viscous string cosmological model in f (r, t) gravity *Eur. Phys. J. Plus* **129** 1

[15] Santos A and Ferst C 2015 Gödel-type solution in f (r, t) modified gravity *Mod. Phys. Lett.* A **30** 1550214

[16] Pradhan A and Chouhan D 2011 Anisotropic bianchi type-i models in string cosmology *Astrophys. Space Sci.* **331** 697–704

[17] Yadav A K, Pradhan A and Singh A K 2012 Bulk viscous lrs bianchi-i universe with variable g and decaying $\lambda$ *Astrophys. Space Sci.* **337** 379–85

[18] Singh C 2013 String cosmology with magnetized bulk viscous fluid in bianchi i universe *Astrophys. Space Sci.* **343** 773–81

[19] Naidu R, Reddy D, Ramprasad T and Ramana K 2013 Bianchi type-v bulk viscous string cosmological model in f (r, t) gravity *Astrophys. Space Sci.* **348** 247–52

[20] Shamir M F 2014 Bianchi type-i cosmology in f (r, t) gravity *J. Exp. Theor. Phys.* **119** 242–50

[21] Singh G *et al* 2015 Bianchi type-i universe with cosmological constant and quadratic equation of state in modified gravity *Adv. High Energy Phys.* **2015**

[22] Kabak A and Aygün S 2020 Scalar field solutions for anisotropic universe models in various gravitation theories *Int. J. Geom. Methods Mod. Phys.* **17** 2050025

[23] Kibble T W 1976 Topology of cosmic domains and strings *J. Phys. A: Math. Gen.* **9** 1387

[24] Yilmaz I 2005 Domain wall solutions in the nonstatic and stationary gödel universes with a cosmological constant *Phys. Rev.* D **71** 103503

[25] Itoh N 1970 Hydrostatic equilibrium of hypothetical quark stars *Prog. Theor. Phys.* **44** 291–2

[26] Bodmer A R 1971 Collapsed nuclei *Phys. Rev.* D **4** 1601

[27] Witten E 1984 Cosmic separation of phases *Phys. Rev.* D **30** 272

[28] Yavuz I, Yilmaz I and Baysal H 2005 Strange quark matter attached to the string cloud in the spherical symmetric space-time admitting conformal motion *Int. J. Mod. Phys.* D **14** 1365–72

[29] Rao V and Sireesha K 2013 Bianchi type-ii, viii & ix cosmological models with strange quark matter attached to string cloud in brans-dicke and general theory of gravitation *Int. J. Theor. Phys.* **52** 1240–9

[30] Chodos A, Jaffe R, Johnson K, Thorn C B and Weisskopf V 1974 New extended model of hadrons *Phys. Rev.* D **9** 3471

[31] Mak M and Harko T 2004 Quark stars admitting a one-parameter group of conformal motions *Int. J. Mod. Phys.* D **13** 149–56

[32] Aziz A, Ray S, Rahaman F, Khlopov M and Guha B 2019 Constraining values of bag constant for strange star candidates *Int. J. Mod. Phys.* D **28** 1941006

[33] Kapusta J I and Landshoff P 1989 Finite-temperature field theory *J. Phys. G: Nucl. Part. Phys.* **15** 267

[34] Pradhan A, Khadekar G, Mishra M K and Kumbhare S 2007 Higher dimensional strange quark matter coupled to the string cloud with electromagnetic field admitting one parameter group of conformal motion *Chin. Phys. Lett.* **24** 3013

[35] Adhav K, Nimkar A, Raut V and Thakare R 2009 Strange quark matter attached to string cloud in bianchi type-iii space time *Astrophys. Space Sci.* **319** 81–4

[36] Khadekar G, Wanjari R and Ozel C 2009 Domain wall with strange quark matter in kaluza-klein type cosmological model *Int. J. Theor. Phys.* **48** 2550–7

[37] Katore S, Rane R and Wankhade K 2010 Domain walls strange quark matter in einstein-rosen space-time with cosmological constant and heat flow *Int. J. Theor. Phys.* **49** 1929–35

[38] Sahoo P K and Mishra B 2013 String cloud and domain walls with quark matter in kink cosmological model *J. Theor. Appl. Phys.* **7** 1–4

[39] Rao V and Sireesha K 2013 Axially symmetric space-time with strange quark matter attached to string cloud in brans-dicke theory of gravitation *Int. J. Theor. Phys.* **52** 1052–60

[40] Sahoo P K and Mishra B 2013 String cloud and domain walls with quark matter for plane symmetric cosmologicalmodel in bimetric theory *J. Theor. Appl. Phys.* **7** 1–5

[41] Rao V and Neelima D 2014 Cosmological models with strange quark matter attached to string cloud in gr and brans-dicke theory of gravitation *Eur. Phys. J. Plus* **129** 1–7

[42] Katore S and Hatkar S 2015 Strange quark matter coupled to string cloud in lyra geometry *Astrophys. Space Sci.* **357** 1–11

[43] Aygün S, Aktaş C, Yìlmaz I and Şahin M 2016 Higher dimensional frw universe solutions with quark and strange quark matter in creation field cosmology *Chin. J. Phys.* **54** 810–9

[44] Agrawal P and Pawar D 2017 Magnetized domain wall in f (r, t) theory of gravity *New Astron.* **54** 56–60

[45] Çağlar H and Aygün S 2017 Higher dimensional frw universe solutions in creation field cosmology *In: AIP Conf. Proc.* vol 1815 (AIP Publishing) (https://doi.org/10.1063/1.4976441)

[46] Özdemir O and Aktaş C 2020 Anisotropic universe models with magnetized strange quark matter in f (r) gravity theory *Mod. Phys. Lett.* A **35** 2050111

[47] Rani N S and Santhi M V 2019 Bianchi type-ii, viii and ix perfect fluid cosmological model with domain walls in f (r, t) gravity *J. Phys.: Conf. Ser.* **1344** 012026

[48] Maurya D C, Pradhan A and Dixit A 2020 Domain walls and quark matter in bianchi type-v universe with observational constraints in f (r, t) gravity *Int. J. Geom. Methods Mod. Phys.* **17** 2050014

[49] Çağlar H 2023 An investigation into lrs bianchi i universe in brans-dicke theory *J. New Theory* 87–96

[50] Singh V, Jokweni S and Beesham A 2023 Plane symmetric cosmological model with strange quark matter in f (r, t) gravity *Universe* **9** 408

[51] Çağlar H, Taşer D and Aygün S 2024 Domain wall with quark matters cosmological models in f (r, t) theory *Indian J. Phys.* **98** 397–405

[52] Tangphati T, Sakalli I, Banerjee A and Pradhan A 2024 Anisotropic quark stars in $f(r,\ l_m,\ t)$ gravity *Chin. Phys. C* (https://doi.org/10.1088/1674-1137/ad99b2)

[53] Jorás S E 2024 Thermodynamics of f (r ) Theories arXiv: 2411.18469

[54] Tangphati T, Sakallı İ, Banerjee A and Pradhan A 2024 Behaviors of quark stars in the rainbow gravity framework *Physics of the Dark Universe* **46** 101610

[55] Júnior J, Moraes P, Brito E and Fortunato J 2024 The palatini formalism of the f (r,l m,t) theory of gravity arXiv: 2411.15615

[56] Tangphati T, Sakallı İ, Banerjee A and Ali A 2024 Interacting quark star with pressure anisotropy and recent astrophysical observations *Chin. J. Phys.* **91** 392–405

[57] Sheikh U, Ramzan N, Aziz Y and Pincak R 2024 Black hole formation from collapsing quark binding string fluid in theory *Gravit. Cosmol.* **30** 441–9

[58] Mangut M, Gürtuğ Ö. and Sakallı İ 2024 On thermal fluctuations and quantum regularities of f(r,g) gravity black holes with constant topological euler density in nonlinear electrodynamics *Phys. Scr.* **99** 055005

[59] Gonçalves J and Santos A 2021 Gödel and gödel-type solutions in the palatini f (r, t) gravity theory *Int. J. Mod. Phys.* D **30** 2150014

[60] Pawar D and Shahare S 2020 Anisotropic tilted cosmological model in f (r, t) theory of gravity *New Astron.* **75** 101318

[61] Pawar D, Mapari R and Pawade J 2021 Perfect fluid and heat flow in f (r, t) theory *Pramana* **95** 10

[62] Abreu E, Marcial M, Mendes A, Oliveira W and Oliveira-Neto G 2012 Noncommutative cosmological models coupled to a perfect fluid and a cosmological constant *J. High Energy Phys.* **2012** 1–23

[63] Bali R and Upadhaya R 2003 Lrs bianchi type i string dust magnetized cosmological models *Astrophys. Space Sci.* **283** 97–108

[64] Shaikh A, Wankhade K and Bhoyar S 2017 Bianchi type-vi0 wet dark fluid cosmological model with linearly varying deceleration parameter in f (r, t) gravity *Int. J. Sci. Res. Sci. Technol.* **3** 1248–54

[65] Mahanta K 2014 Bulk viscous cosmological models in f (r, t) theory of gravity *Astrophys. Space Sci.* **353** 683–9

[66] Jokweni S, Singh V and Beesham A 2021 Lrs bianchi i model with bulk viscosity in gravity *Gravit. Cosmol.* **27** 169–77

[67] Collins C, Glass E and Wilkinson D 1980 Exact spatially homogeneous cosmologies *Gen. Relativ. Gravit.* **12** 805–23

[68] Zel'dovich Ya B 1972 A hypothesis, unifying the structure and entropy of the universe *Mon. Not. R. Astr Soc.* **160**

[69] Chavanis P-H 2015 Cosmology with a stiff matter era *Phys. Rev.* D **92** 103004

[70] Mathew T K, Aswathy M and Manoj M 2014 Cosmology and thermodynamics of flrw universe with bulk viscous stiff fluid *Eur. Phys. J* C **74** 1–10

[71] Shamir M F, Ahmad Z and Raza Z 2015 Gravitational dust collapse in f (r) gravity *Int. J. Theor. Phys.* **54** 1450–60

[72] Taşer D 2019 f (r) gravitasyon teorisinde toz bulutlu konformal düz uzay-zamanlar *Süleyman Demirel Üniversitesi Fen Bilimleri Enstitüsü Dergisi* **23** 59–65

[73] Yilmaz Ä 2006 String cloud and domain walls with quark matter in 5-d kaluza-klein cosmological model *Gen. Relativ. Gravit.* **38** 1397–406

[74] Katore S and Hatkar S 2016 Bianchi type iii and kantowski-sachs domain wall cosmological models in the f (r, t) theory of gravitation *Progress of Theoretical and Experimental Physics* **2016** 033–01

[75] Santos J, Alcaniz J, Reboucas M and Carvalho F 2007 Energy conditions in f (r) gravity *Phys. Rev.* D **76** 083513

[76] Samanta G C and Godani N 2019 Validation of energy conditions in wormhole geometry within viable f (r) gravity *Eur. Phys. J* C **79** 1–11

[77] Alvarenga F G, Houndjo M J S, Monwanou A V and Orou J B C 2012 Testing some f (r, t) gravity models from energy conditions arXiv: 1205.4678

[78] Bishi B K, Beesham A and Mahanta K L 2022 Cosmology in f (r, t) gravity with quadratic deceleration parameter *Zeitschrift für Naturforschung* A **77** 259–68