

BSN. IV. The First Multiband Light Curve Study of Five W UMa-type Contact Binary Systems

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Abstract

In this work, we present a detailed investigation of five contact binary systems of the W Ursae Majoris (W UMa) type. Multiband photometric observations were conducted using ground-based telescopes in both the northern and southern hemispheres, yielding new times of minima. $O - C$ diagram analysis reveals that two systems exhibit parabolic trends, indicating a gradual long-term decrease in their orbital periods. The light curves were modeled using version 1.0 of the BSN application, with one system requiring the inclusion of a cool starspot to achieve a satisfactory fit. We examined empirical relationships between orbital period and fundamental parameters, identifying the period–semimajor axis (P – a) relation as the most robust correlation, which was used to estimate absolute parameters. To statistically assess thermal equilibrium, we analyzed temperature differences between components and found that 90% of systems exhibit less than 9.4% contrast. Two target systems with extremely low mass ratios were identified, and their orbital stability was evaluated. Based on the effective temperatures and component masses, two systems were classified as W-subtype and three as A-subtype. The evolutionary status of the binaries was assessed through their locations in mass–radius, mass–luminosity, and other empirical diagrams, and initial component masses as well as total mass loss were also estimated.

Supplementary material for this article is available [online](#)

Unified Astronomy Thesaurus concepts: [Eclipsing binary stars \(444\)](#); [Fundamental parameters of stars \(555\)](#); [Astronomy data analysis \(1858\)](#)

1. Introduction

Contact binaries are an important class of close binary stars, consisting of two stellar components that share a common envelope, enabling the exchange of both mass and energy between them (Lucy 1968a, 1968b). These systems are predominantly composed of late-type stars, including spectral types F, G, K, and occasionally M, and they represent a considerable portion of the stellar population. Estimates suggest that roughly one in every 500 main-sequence stars in our Galaxy is part of a contact binary (Rucinski 2007). Contact binaries are further categorized into A-subtype and W-subtype

systems, distinguished by which component is hotter: the more massive star in A-subtype systems or the less massive one in W-subtype systems (Binnendijk 1970). Despite substantial research, the evolutionary connections between these subtypes remain a matter of debate (Zhang et al. 2020).

A widely accepted scenario proposes that contact binaries originate from short-period detached binaries that evolve into contact configurations through angular momentum loss driven by magnetic braking (Vilhu 1982; Stepień 2006; Qian et al. 2017). Over time, these systems may merge into rapidly rotating single stars, potentially giving rise to objects like blue stragglers or FK Com-type stars (Rasio 1995). However, direct observational evidence for such mergers is limited, with V1309 Sco being one of the few well-documented examples (Tylenda et al. 2011).



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The distribution of orbital periods among contact binaries typically peaks near 0.27 day, while a well-established short-period cutoff around 0.22 day suggests a physical limit potentially imposed by angular momentum loss processes or structural instabilities in low-mass components (Jiang et al. 2012). Moreover, variations in orbital periods are strongly linked to mass transfer dynamics and angular momentum loss mechanisms such as magnetic braking (Qian et al. 2013).

A characteristic phenomenon observed in many contact binaries is the O’Connell effect, where the two maxima in their light curves show unequal brightness. This asymmetry is often linked to magnetic activity and starspots on the stellar surfaces, though other explanations such as hot spots caused by mass transfer or the presence of circumstellar material have also been suggested (O’Connell 1951; Liu & Yang 2003). Clarifying the origin of this effect is crucial for accurately determining the physical parameters of these systems, as surface inhomogeneities can significantly impact photometric analyses (Liu 2021).

In this work, ground-based multiband photometric observations were conducted for five eclipsing contact binary systems of the W Ursae Majoris (W UMa) type, aiming to refine their orbital and physical parameter estimates. This research extends the efforts begun by Poro et al. (2025b, 2025c, 2025d), offering new observations and analyses focused on additional W UMa-type contact binary stars within the framework of the BSN project.¹⁰ The paper is structured as follows: Section 2 introduces the target systems. Section 3 describes the acquisition and reduction of both ground- and space-based photometric data. Variations in orbital periods are analyzed in Section 4. Section 5 presents the solutions to the photometric light curves of the targets. The determination of absolute parameters is detailed in Section 6. Finally, Section 7 discusses the findings and summarizes the conclusions.

2. Target Binary Stars

We have analyzed five eclipsing binary stars, including BF Dor, CRTS J014306.6+383909 (hereinafter J014306), LZ Leo, WISE J201317.7+464036 (hereinafter J201317), and ZTF J004331.79+500059.9 (hereinafter J004331). These contact binary systems had not been studied in detail previously. Also, we had multiband photometric data available in the BSN project database for these systems, providing sufficient observational coverage for accurate analysis. Table 1 presents specifications for the target systems based on the Gaia DR3 database (Gaia Collaboration et al. 2023). The general properties of the target systems are summarized below:

1. *BF Dor*. According to the Variable Star Index (VSX) report, this system was discovered by Friedhelm Hund. In both the VSX database and the General Catalog of Variable Stars (GCVS; Samus’ et al. 2017), BF Dor is classified as a contact binary system with an orbital period of 0.352271 day. However, in the All-Sky Automated Survey for Supernovae (ASAS-SN; Jayasinghe et al. 2018) and the Zwicky Transient Facility (ZTF; Sánchez-Sáez et al. 2023) catalogs, it is listed as an EA-type (detached) system with a period of 0.70455 day; twice the value of GCVS. The VSX reports a maximum apparent magnitude of 13^{mag} for this system.

2. *J014306*. This binary system was discovered in the Catalina Surveys Data Release 1 (CSDR1; Drake et al. 2014). The CSDR1, VSX, ZTF, and ASAS-SN catalogs consistently classify J014306 as a contact binary system with an orbital period of 0.27923 day. The VSX database reports an apparent magnitude of 14.30^{mag} for this system.

3. *LZ Leo*. This system was discovered in the CSDR1 catalog. LZ Leo is classified as a contact binary star in the VSX, ASAS-SN, ZTF, and GCVS catalogs. The VSX and GCVS databases report maximum apparent magnitudes of 13.19^{mag} and 12.97^{mag} for this system, respectively.

4. *J201317*. This system is classified as a contact binary in the Wide-field Infrared Survey Explorer (WISE; Chen et al. 2018) catalog. Limited information about J201317 is available in existing catalogs. Its orbital period is listed as 0.2967596 day in the VSX database. The effective temperature reported in the TESS Input Catalog (TIC) is 5143 ± 195 K, which is consistent with the temperature derived from Gaia DR3.

5. *J004331*. This system was discovered by the Asteroid Terrestrial-impact Last Alert System (Tonry et al. 2018) catalog. J004331 is reported as a contact binary in the VSX, ASAS-SN, and ZTF catalogs, with an orbital period of 0.392775 day. The VSX database lists its apparent magnitude as 13.373(168)^{mag} in *r* passband.

3. Observation and Data Reduction

Photometric observations and data reduction for the five target binary systems were carried out using standard filters at three observatories located in both the Northern and Southern Hemispheres: Complejo Astronómico El Leoncito (CASLEO), San Pedro Mártir (SPM), and the Observatoire Astronomique des Binaires André Coliac (OABAC). Table 2 provides details for each system, including the observation date, type of filter, exposure time (in seconds), the maximum apparent magnitude of the systems in our observations in filter V, and the observatory where the data were obtained. The general characteristics of the comparison and check stars used during the observations and data reductions are shown in Table 3. These stars played a critical role in ensuring the precision and stability of the photometric measurements. The use of

¹⁰ <https://bsnp.info/>

Table 1
Specifications of the Target Systems From Gaia DR3

System	R.A. (deg) (J2000)	Decl. (deg) (J2000)	d (pc)	RUWE	T (K)
BF Dor	91.477281	−66.843424	615(6)	1.146	5636(10)
CRTS J014306.6+383909 (J014306)	25.778003	38.652536	525(6)	1.022	4921(6)
LZ Leo	149.339006	14.204328	403(3)	1.000	...
WISE J201317.7+464036 (J201317)	303.323961	46.676887	943(21)	1.014	5259(46)
ZTF J004331.79+500059.9 (J004331)	10.882479	50.016630	1349(47)	2.135	5259(19)

Table 2
Specifications of the Ground-based Photometric Observations, Including the Observation Dates, Filters, Exposure Times (in seconds), Maximum Apparent Magnitudes in the V Band, and Observatory Sites For Each Target System

System	Observation Date	Filter	Exposure Time (s)	$V_{\max}$ (mag)	Observatory
BF Dor	2024 Dec 27–2025 Feb 22	BVR_cI_c	$B(30), V(15), R_c(10), I_c(12)$	13.08(7)	CASLEO
J014306	2024 Oct 15	BVR_cI_c	$B(90), V(50), R_c(35), I_c(30)$	14.25(8)	SPM
LZ Leo	2024 Apr 4, 11	BVR_c	$B(120), V(120), R_c(120)$	13.20(5)	OABAC
J201317	2024 Jul 23, and Jul 31	BVR_cI_c	$B(90), V(50), R_c(35), I_c(30)$	13.02(10)	SPM
J004331	2024 Sep 10, 15, 20, and Oct 10	VR_c	$V(180), R_c(180)$	13.65(11)	OABAC

Table 3
List the Comparison and Check Stars in the Ground-based Observations

System	Star Type	Star Name	R.A. (deg) (J2000)	Decl. (deg) (J2000)
BF Dor	Comparison	UCAC4 116-011637	91.378580	−66.844710
BF Dor	Check	2MASS 06054182-6650129	91.424289	−66.836978
J014306	Comparison	2MASSJ01433748+3836044	25.906183	38.601202
J014306	Check	2MASSJ01431770+3838529	25.823790	38.648055
LZ Leo	Comparison	Gaia DR2 615285229435410432	149.313674	14.246790
LZ Leo	Check	Gaia DR2 615661679023766528	149.287898	14.305030
J201317	Comparison	2MASSJ20131800+4642445	303.325019	46.712383
J201317	Check	2MASSJ20133198+4641317	303.383282	46.692186
J004331	Comparison	Gaia DR2 414729124506961152	10.737472	50.063308
J004331	Check	Gaia DR2 414729468104365056	10.850449	50.034823

comparison and check stars helped minimize systematic errors and improve the accuracy of the resulting light curves.

3.1. CASLEO Observatory

Observations of BF Dor were carried out with the 2.15 m Jorge Sahade (JS) telescope at the CASLEO Observatory in Argentina (located at 69°18'W, 31°48'S, 2552 m elevation). Data acquisition employed a Versarray 2048B CCD camera (Roper Scientific, Princeton Instruments) in combination with standard BVR_cI_c filters. The detector provided a plate scale of $0''.15 \text{ pixel}^{-1}$, and observations were performed using a 5×5 binning mode. CCD data reduction and aperture photometry were conducted with the APPHOT package within the Image Reduction and Analysis Facility (IRAF; Tody 1986), utilizing bias frames and flat-field corrections.

3.2. SPM Observatory

The binary systems J014306 and J201317 were observed at the SPM Observatory in México, situated at longitude 115°27'49"W, latitude 31°02'39"N, and an altitude of 2830 m. Observations were performed using two Ritchey-Chrétien telescopes. The 0.84 m telescope, operating at an $f/15$ focal ratio, was equipped with the Mexman filter wheel and a Marconi 5 CCD camera (e2v CCD231-42), featuring $15 \times 15 \mu\text{m}^2$ pixels, a gain of $2.2 e^-/\text{ADU}$, and a readout noise of $3.6 e^-$. The 1.5 m telescope employed the RUCA filter wheel along with the Spectral Instruments 1 detector, housing an e2v CCD42-40 chip with $13.5 \times 13.5 \mu\text{m}^2$ pixels, a gain of $1.39 e^-/\text{ADU}$, and a readout noise of $3.49 e^-$. Standard B , V , R_c , and I_c filters were used for these observations. Data reduction and photometry were carried out with IRAF software

Table 4
Specifications of the TESS Data Used in this Study

System	TIC	TESS Sector	Observation Year	Exposure Length	Error Average
BF Dor	41169654	2,5,8,9,12,29,32, 35,39,62,63,64,65, 66,67,68,69,89,90	2018,2018,2019,2019,2019,2020,2020, 2021,2021,2023,2023,2023,2023, 2023,2023,2023,2023,2025,2025	1800,1800,1800,1800,1800,600,600, 600,600,200,200,200,200, 200,200,200,200,200,200	0.0391
LZ Leo	358669926	45,46,72	2021,2021,2023	600,600,200	0.1874
J004331	240745396	17,57,58,84,85	2019,2022,2022,2024,2024	1800,200,200,200,200	0.0519

tools, following standard procedures such as bias subtraction and flat-field correction, as described by Tody (1986).

3.3. OABAC Observatory

The OABAC in Marseille, France, located at longitude $05^{\circ}27'56''\text{E}$ and latitude $43^{\circ}18'54''\text{N}$. For LZ Leo, a 200 mm Newtonian telescope with an $f/4$ focal ratio was used, paired with an ASI ZWO 533MM Pro CCD camera and standard BVR_c filters. J004331 was observed with a 150 mm Newtonian telescope, also at $f/4$, coupled with an ASI ZWO 183MM Pro CCD and standard VR_c filters. Both setups incorporated a field corrector during observations. Data were acquired using a 4×4 binning mode. The average CCD operating temperatures were -10°C for LZ Leo and 0°C for J004331. Preprocessing and standard data reduction, including dark, bias, and flat-field corrections, were carried out with Muniwin 2.1.35, Siril,¹¹ and Prism v.10 software.

3.4. TESS Observations

NASA launched the Transiting Exoplanet Survey Satellite (TESS) in 2018 to search for exoplanets across the sky (Ricker et al. 2010; Stassun et al. 2018). Equipped with four wide-field cameras, TESS observes different regions of the sky, dedicating about 27.4 days to each sector. In this study, we used time-series data from TESS for the binary systems BF Dor, LZ Leo, and J004331. The TESS light curves are provided in the “TESS:T” passband, which covers a broad wavelength range of 600–1000 nm (Ricker et al. 2015). The TESS sectors used in this work are listed in Table 4. All TESS data were obtained from the Mikulski Archive for Space Telescopes (MAST).¹² We extracted the light curves using the Lightkurve software package¹³ and applied detrending based on the TESS Science Processing Operations Center (SPOC) pipeline (Jenkins et al. 2016).

4. Investigating Orbital Period Variations

Conducting further analysis of these targets required gathering as many eclipse timing measurements as possible

from photometric surveys, including both ground-based and space-based observations. Using the available eclipse timings, we analyzed the orbital period variations of all five targets. However, for J201317, only our own observed minima were available, and the limited number of eclipsing times prevented any meaningful further analysis. Moreover, eclipse timing data for LZ Leo were gathered from the VarAstro database,¹⁴ where no uncertainties are reported for the available measurements. Eclipse timings were directly determined from the TESS 2 and 10 minutes cadence data. In contrast, for the more sparsely sampled 30 minutes cadence data, we first applied the phase-folding approach outlined by Li et al. (2020) to align the observations within a single orbital cycle. Once aligned, eclipse timings were extracted. The minima times are commonly extracted using the Kwee & van Woerden (1956) (KW) method, which calculates mid-eclipse times from binary system light curves. Although widely used, the KW method tends to produce underestimated error values (Pribulla et al. 2012; Li et al. 2018). Moreover, its performance can be inadequate for asymmetric or incomplete light curves (Mikulášek et al. 2013). To address these issues, we fitted Gaussian and Cauchy distribution models, based on Poro et al. (2021) approach, to selected portions of the light curves containing the minima. Uncertainties were estimated via MCMC sampling, implemented in Python using the emcee package (Foreman-Mackey et al. 2013).

To ensure consistency in time measurements across all data sets, we converted Heliocentric Julian Dates (HJD) to Barycentric Julian Dates in Barycentric Dynamical Time (BJD_{TDB}) using an online conversion tool.¹⁵ This step was necessary because our data included both time formats. The eclipse timings derived from our observations are presented in Table 5. A machine-readable version of the extracted and compiled eclipse timing data for the target binary systems is available. Next, we computed the $O - C$ values based on the reference ephemeris:

$$\text{BJD} = \text{BJD}_0 + P \times E, \quad (1)$$

where BJD represents the observed eclipse timing, BJD_0 (given in the second column of Table 6) denotes the initial

¹¹ <https://siril.org/>

¹² <https://mast.stsci.edu/portal/Mashup/Clients/Mast/Portal.html>

¹³ <https://docs.lightkurve.org>

¹⁴ <http://var.astro.cz>

¹⁵ <https://astroutils.astronomy.osu.edu/time/hjd2bjd.html>

Table 5
The Times of Minima Extracted from Our Ground-based Observations

System	Min. (BJD _{TDB})	Error	Epoch	$O - C$
BF Dor	2460671.6576	0.0032	−0.5	0.0038
	2460671.8299	0.0028	0	0
J014306	2460598.7223	0.0049	−1	0.0002
	2460598.8597	0.0054	−0.5	−0.0021
	2460599.0014	0.0041	0	0
LZ Leo	2460405.3129	0.0026	0	0
	2460405.4552	0.0029	0.5	−0.0005
	2460412.4089	0.0029	24	−0.0004
J201317	2460514.8709	0.0009	0	0
	2460522.7348	0.0006	26.5	−0.0002
	2460522.8843	0.0007	27	0.0009
J004331	2460564.3977	0.0015	−25.5	0.0062
	2460569.5024	0.0019	−12.5	0.0048
	2460574.4073	0.0016	0	0
	2460594.4411	0.0011	51	0.0023
	2460594.6346	0.0018	51.5	−0.0006

primary eclipse time, and P (provided in the third column of Table 6) corresponds to the orbital period. The calculated epochs and $O - C$ values are provided in Table 5 and are also available in an online machine-readable format. The corresponding $O - C$ diagrams are shown in Figure 1.

We find that two of the targets, BF Dor and J014306, exhibit linear trends, while LZ Leo and J004331 display parabolic variations. For the targets with parabolic behavior, we applied the following equation for the $O - C$ fitting:

$$O - C = \Delta T_0 + \Delta P_0 \times E + \frac{\beta}{2} E^2. \quad (2)$$

The revised ephemerides are presented in Table 6. For the two systems displaying parabolic variations, the corresponding fitted parameters and estimated mass transfer rates are detailed in Table 7. We found that the LZ Leo and J004331 systems exhibit a long-term decrease.

5. Light Curve Solution

The BSN application version 1.0 (Paki et al. 2025), specifically developed for modeling contact binary stars, was used to analyze the photometric light curves of the target systems. This application offers an expanded feature set and a more intuitive interface while adhering to established scientific standards. Currently, it only supports the Windows operating system.

For the target systems, the contact configuration was selected, as the observed light curve shapes, catalog classifications, and their short orbital periods all indicate that these systems are in physical and thermal contact. Time was converted to phase for the light curves using ephemerides in Table 6. The gravity-darkening coefficient was fixed at $g_1 = g_2 = 0.32$ following Lucy (1967), and the bolometric

albedo was set to $A_1 = A_2 = 0.5$ as per Ruciński (1969). The stellar atmosphere model adopted was that of Castelli & Kurucz (2004). In the BSN application, linear and logarithmic limb-darkening formulations are implemented using coefficients adopted from the tabulations provided by van Hamme (1993).

For the analysis, the initial effective temperatures (T) were obtained from the Gaia DR3 database (Table 1). The LZ Leo system lacked a temperature entry in Gaia DR3; for this case, the temperature of 5113(305) K from version 8.2 of the TIC was used instead. For LZ Leo, an independent spectroscopic estimate from the LAMOST-Low Resolution Spectroscopy survey (5145 ± 38 K) is also available, which is in good agreement with the TIC value, supporting the reliability of the adopted temperature. It was assumed that the temperatures reported in Gaia DR3 and TIC correspond to the hotter components of each system, as indicated by the depths of the light curve minima. The effective temperature of the cooler star was estimated based on the difference in depth between the primary and secondary minima in the light curves.

For each system, the mass ratio (q)-search method was employed to estimate the initial mass ratio and its possible range, which served as the starting point for the light curve analysis (Terrell & Wilson 2005). A wide range of q values, from 0.05 to 20, was examined for all target systems. Subsequently, a narrower range was explored to refine the estimate by minimizing the sum of squared residuals between the observed and synthetic light curves. As illustrated in Figure 2, each q -search curve shows a clear minimum in the residual sum, indicating the preliminary mass ratio. This initial estimate serves as the starting point for further analysis, with the final mass ratio obtained through iterative light curve modeling techniques. It is worth noting that, according to Poro et al. (2024c), photometric analyses of systems with different orbital inclinations can still achieve reliable q -search accuracy when iterative methods such as MCMC are employed. Similar investigations have also been presented by Liu et al. (2023) and Li et al. (2021) in this regard.

An asymmetry between the light curve maxima, known as the O’Connell effect, was observed only in the J014306 system. To achieve an acceptable theoretical fit for this system, a cool starspot on the primary component was required (Table 8). This effect is commonly attributed to magnetic activity on the stellar surface, which leads to the formation of starspots (O’Connell 1951). Although this explanation is widely accepted, other physical interpretations have also been proposed to explain the phenomenon more comprehensively, such as those suggested by Zhou & Leung (1990) and Liu & Yang (2003).

We used the photometric multiband data and initial parameter values to achieve a satisfactory theoretical fit. The optimization tool in the BSN application was subsequently employed to improve the light curve solution, providing better-

Table 6
Reference and New Ephemeris of the Five Systems

System	Reference ephemeris		New Ephemeris	
	t_0 (BJD _{TDB})	Period (day)/Source	Corrected t_0 (BJD _{TDB})	New Period (days)
BF Dor	2460671.8299(8)	0.352271/VSX	2460671.8319(3)	0.35227839(6)
J014306	2460599.0014(11)	0.279231/ASAS-SN	2460599.0008(5)	0.27923397(10)
LZ Leo	2460405.3129(26)	0.295682/VSX	2460405.3149(2)	0.29567992(6)
J201317	2460514.8709(9)	0.2967596/VSX	...	...
J004331	2460574.4073(16)	0.3927752/VSX	2460574.4102(2)	0.3927729(3)

Note. The reference times of minimum (t_0) were obtained from our observations in this study.

constrained parameter estimates compared to the initial values, including the effective temperatures, mass ratio, fillout factor, and orbital inclination. In the single case where a starspot was included in the model, the optimization process also provided a refined determination of its position.

To derive the final solutions and estimate parameter uncertainties, we employed the Markov Chain Monte Carlo (MCMC) method. The BSN application offers substantially higher computational performance in MCMC fitting, generating synthetic light curves over 40 times faster than PHOEBE Python code version 2 (Paki et al. 2025). This improvement is primarily due to BSN’s optimized architecture and the integration of modern computational libraries, while the fundamental approaches for light curve analysis remain consistent with those used in other established binary star modeling tools. In the MCMC simulations, we used 24 walkers and 2000 iterations to sample five main parameters ($T_{1,2}$, q , f , and i), providing both their estimated values and corresponding uncertainties.

The corner plots for the target systems are shown in Figure 3, providing a visualization of the parameter distributions and correlations from the MCMC analysis. Table 8 presented the results of the light curve modeling, including the estimated parameters and their uncertainties. Figure 4 illustrates the final synthetic light curves overlaid on the observed data for the binary systems. Additionally, three-dimensional (3D) visualizations of the binary systems are presented in Figure 5.

6. Absolute Parameters

6.1. Method

Accurate determination of absolute parameters plays an important role in studying the evolution of contact systems and exploring the correlations among their physical properties. Several approaches are employed to derive the absolute parameters of contact binaries. One such method, applicable when only photometric data are available, relies on Gaia DR3 parallaxes, as comprehensively discussed by Poro et al.

(2024b). This technique depends significantly on two factors: the interstellar extinction A_V and the maximum visual magnitude $V_{\max}$. The precision of $V_{\max}$ is linked to the quality of observational procedures, whereas high values of A_V can hinder achieving reliable accuracy in absolute parameter estimation using Gaia DR3 parallaxes. In this study, we derived A_V from the three-dimensional dust map corresponding to Gaia distances (Green et al. 2019). Consequently, for J201317 system, the A_V values (Table 9) is too large to allow the use of Gaia DR3 parallaxes for reliable parameter estimation. As we aimed to apply a consistent method for target systems, we therefore needed to use a different way.

Another way to estimate absolute parameters from photometric data in contact binary systems is to use empirical parameter relationships. Various studies have proposed such relationships, particularly those describing the connection between orbital period and mass (P – M), as well as between orbital period and semimajor axis (P – a). However, the P – M relationships are usually reliable only for the more massive primary component, while significant scatter is observed in the corresponding plots for the secondary components (Poro et al. 2022). However, it is also essential to perform a statistical analysis of the scatter and the strength of the correlations between empirical parameters in order to identify the most suitable empirical parameter relationship for estimating absolute parameters.

The empirical relationships between parameters in contact binary systems remain subject to debate, partly due to diverse analysis techniques and limited sample sizes (e.g., Poro et al. 2022, 2024d). In this study, we focus on the relationships between the orbital period (P) and key physical parameters, including mass (M), radius (R), and luminosity (L), for each component. Using the sample presented by Poro et al. (2025a), which initially included 818 systems, we selected only those with available data for P , M , R , and L , resulting in a final sample of 483 systems for analysis. Our goal is to identify the strongest statistically significant relationships to select the most reliable ones for estimating absolute parameters.

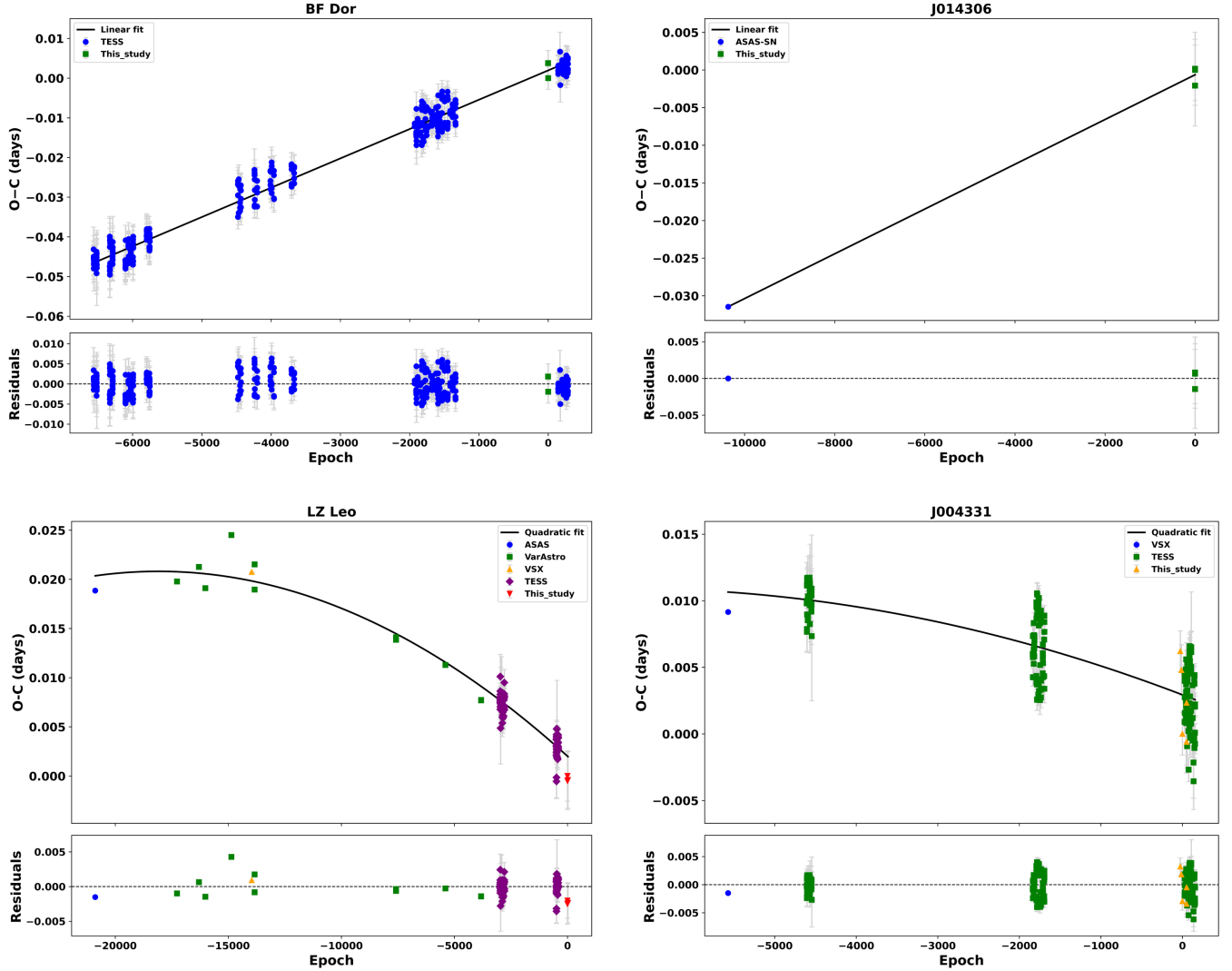
Figure 1. The $O - C$ diagrams of the target systems, with residuals at the bottom.

Table 7
The $O - C$ Fitting Coefficients and Mass Transfer Rate

Parameter	ΔT_0 ($\times 10^{-4}$ days)	Error	ΔP_0 ($\times 10^{-7}$ days)	Error	β ($\times 10^{-7}$ days yr $^{-1}$)	Error	dM_1/dt ($\times 10^{-7} M_\odot$ yr $^{-1}$)	Error
LZ Leo	20.1	1.4	-20.8	0.6	-1.4	0.1	2.2	3.1
J004331	29.4	2.3	-23.4	2.9	-3.2	1.2	-0.4	0.4

The correlation coefficients between the orbital period and selected parameters were computed and presented as a horizontal bar plot (Figure 6, top panel). As the histogram illustrates only positive correlation values, negative correlations are not included in this analysis. The length of each bar represents the strength of the positive linear relationship between the period and the respective parameter. To further investigate these associations, scatter plots of the period versus

each parameter are provided in the bottom panel of the same figure (Figure 6, bottom panel). Together, these visualizations offer a comprehensive overview of the magnitude and nature of the dependencies within the data set.

Based on the results, the relationship between the orbital period and the semimajor axis shows the strongest correlation coefficient of 0.90. These numerical values are also indicated on the histogram bars for clarity.

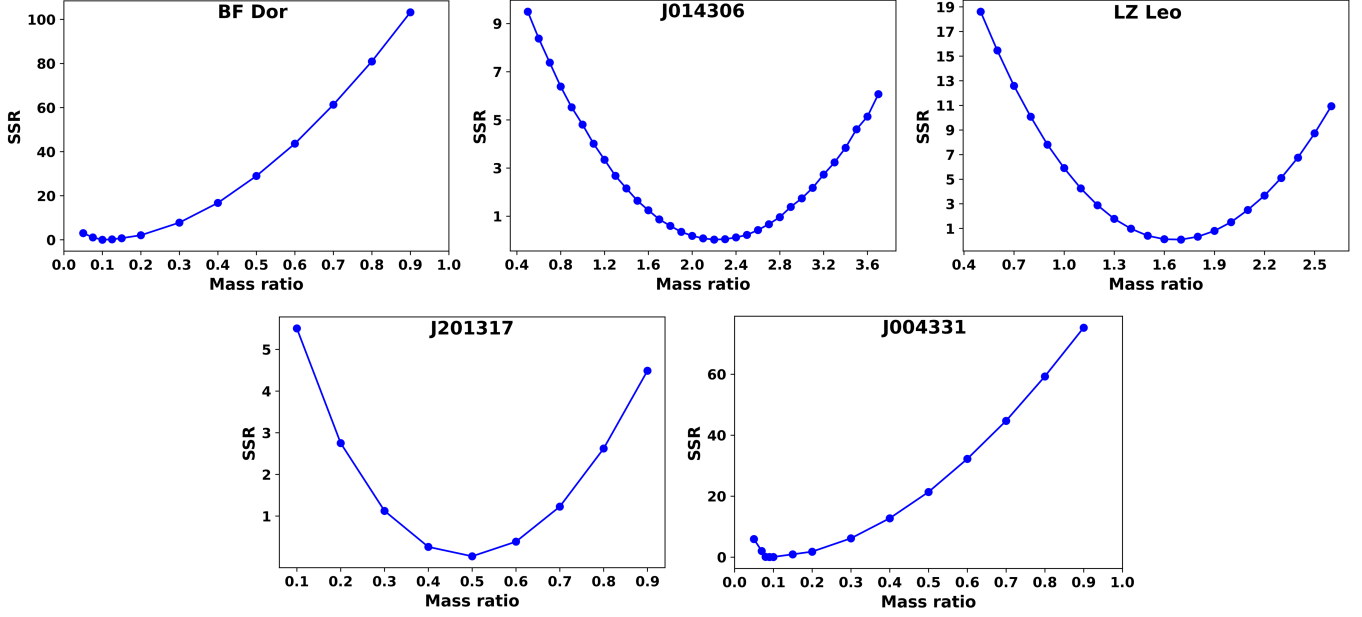


Figure 2. The sum of squared residuals as a function of mass ratio.

Table 8
Light Curve Solutions of the Target Binary Stars

Parameter	BF Dor	J014306	LZ Leo	J201317	J004331
T_1 (K)	5534^{+100}_{-107}	5325^{+17}_{-17}	5187^{+86}_{-87}	5234^{+82}_{-81}	5401^{+104}_{-97}
T_2 (K)	4754^{+101}_{-105}	4736^{+17}_{-16}	4929^{+75}_{-77}	5148^{+81}_{-77}	5091^{+106}_{-105}
$q = M_2/M_1$	0.100^{+24}_{-23}	2.232^{+120}_{-134}	1.665^{+134}_{-173}	0.487^{+130}_{-56}	0.093^{+41}_{-29}
i°	$50.94^{+1.98}_{-1.92}$	$87.71^{+1.66}_{-1.74}$	$85.69^{+1.89}_{-1.41}$	$83.01^{+3.27}_{-2.45}$	$60.54^{+3.91}_{-2.82}$
f	0.229^{+100}_{-98}	0.169^{+55}_{-54}	0.066^{+48}_{-38}	0.161^{+86}_{-79}	0.126^{+92}_{-73}
$\Omega_1 = \Omega_2$	1.945(33)	5.475(302)	4.731(293)	2.803(146)	1.930(10)
$l_1/l_{\text{tot}}(V)$	0.935(11)	0.458(5)	0.445(7)	0.675(9)	0.918(7)
$l_2/l_{\text{tot}}(V)$	0.065(3)	0.542(5)	0.555(7)	0.325(7)	0.082(2)
$r_{\text{(mean)1}}$	0.588(7)	0.322(12)	0.339(5)	0.454(11)	0.591(3)
$r_{\text{(mean)2}}$	0.213(8)	0.462(11)	0.429(6)	0.329(13)	0.205(4)
Component	...	Primary	...	...	...
Col. $^\circ$ (spot)	...	96(2)	...	...	...
Long. $^\circ$ (spot)	...	312(3)	...	...	...
Radius $^\circ$ (spot)	...	20(1)	...	...	...
$T_{\text{spot}}/T_{\text{star}}$	...	0.73(1)	...	...	...

6.2. Parameter Estimation

A revised empirical relationship between P and a for contact binary systems was presented by Poro et al. (2024e). This calibration was derived using observations of 414 systems compiled in the work of Latković et al. (2021), all exhibiting periods shorter than 0.7 day. The relation is expressed as follows (Equation (3)):

$$a = (0.372^{+0.113}_{-0.114}) + (5.914^{+0.272}_{-0.298}) \times P, \quad (3)$$

where a is measured in solar radii ($R_\odot$) and P in days.

Since the P – a relationship exhibits a stronger correlation compared to the relationships between the orbital period and other parameters such as mass, radius, and luminosity, we adopted this empirical relation for estimating the absolute parameters. The procedure began with Equation (3), in which the orbital period of each target system, was used to calculate the semimajor axis. The stellar masses were then determined using the mass ratio derived from the light curve solutions in

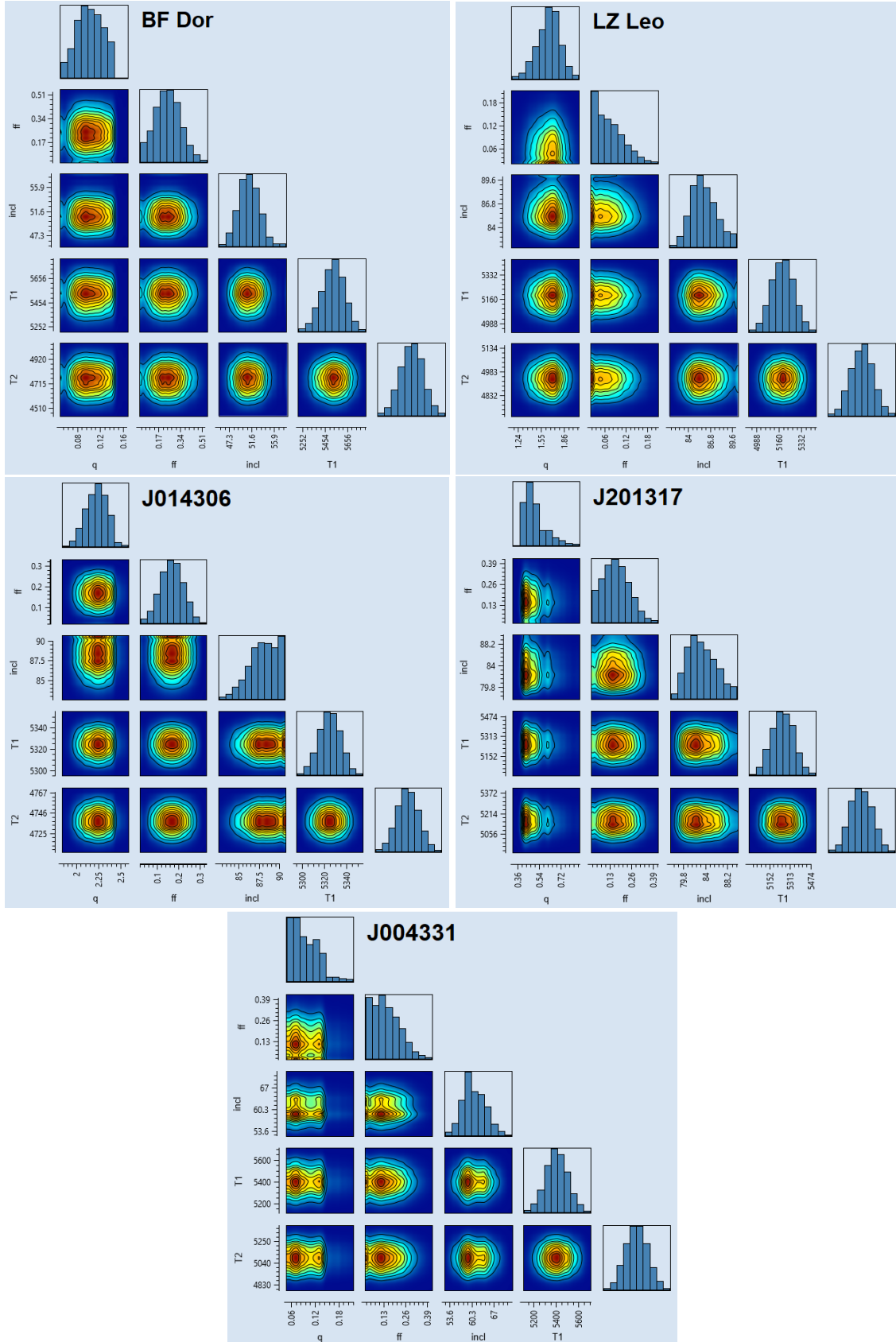


Figure 3. The corner plots of the target systems were determined by MCMC modeling.

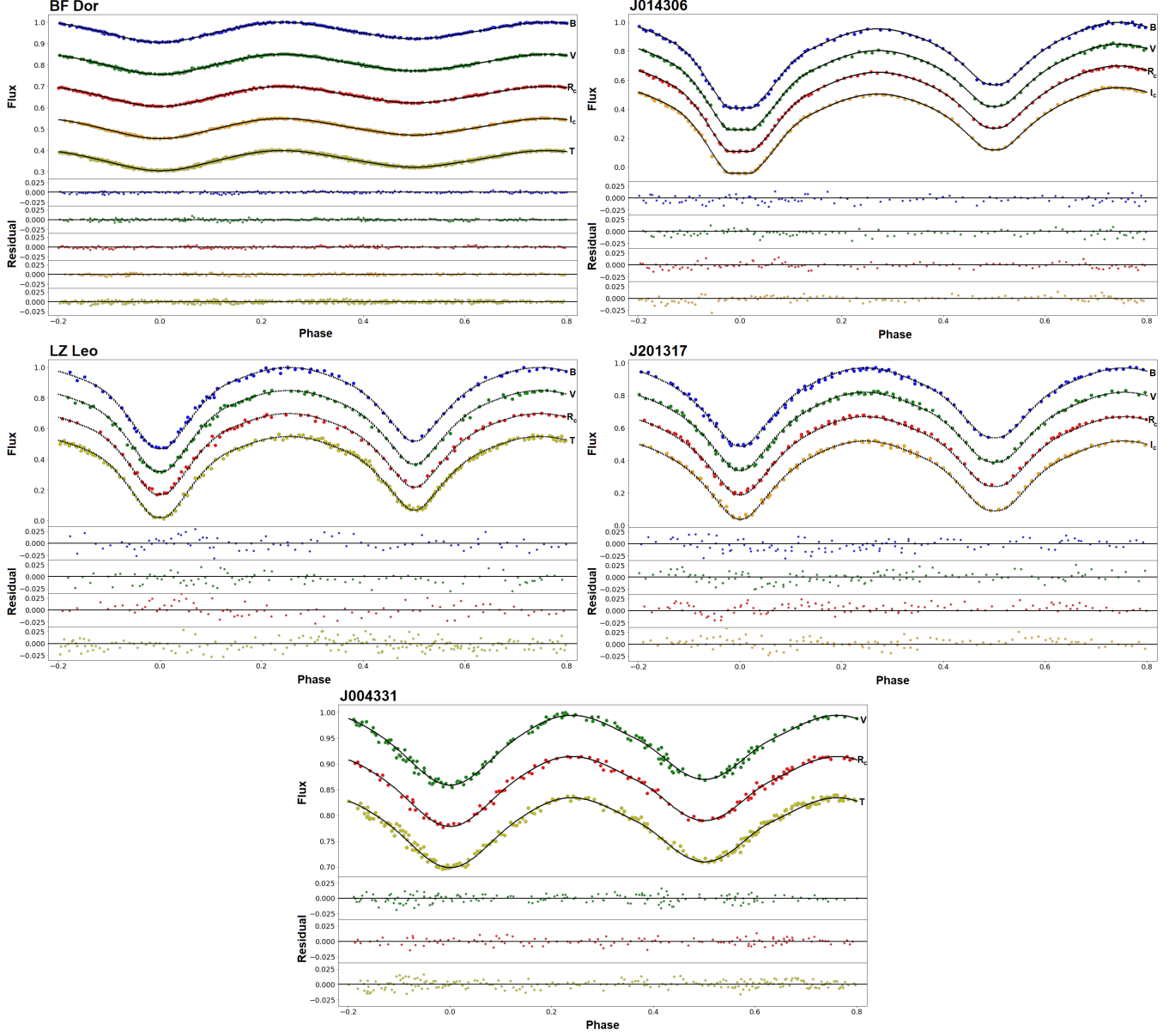


Figure 4. The colored dots represent the observed light curves of the systems in different filters, and the synthetic light curves, generated using the light curve solutions, are also shown. Residuals are shown at the bottom of each panel.

combination with Kepler's third law, as expressed by Equations (4) and (5).

$$M_1 = \frac{4\pi^2 a^3}{GP^2(1+q)}, \quad (4)$$

$$M_2 = q \times M_1. \quad (5)$$

The radii (R) of the stellar components were calculated using the mean fractional radii ($r_{\text{mean}, 1}$ and $r_{\text{mean}, 2}$) provided in Table 8, applying the relation $R_{1,2} = a \times r_{\text{mean}, 1,2}$. With the effective temperature and radius determined, the stellar luminosities were subsequently computed. The absolute

bolometric magnitude (M_{bol}) for each component was then derived from the luminosity by employing the standard relation between these parameters, as given in Equation (6):

$$M_{\text{bol}, 1,2} = M_{\text{bol}, \odot} - 2.5 \times \log\left(\frac{L_{1,2}}{L_{\odot}}\right). \quad (6)$$

In Equation (6), the absolute bolometric magnitude of the Sun is taken as 4.73^{mag} , following the value reported by Torres (2010). The surface gravity (g) of each star was computed on a logarithmic scale using the determined masses and radii. Furthermore, the orbital angular momentum (J_0) was derived

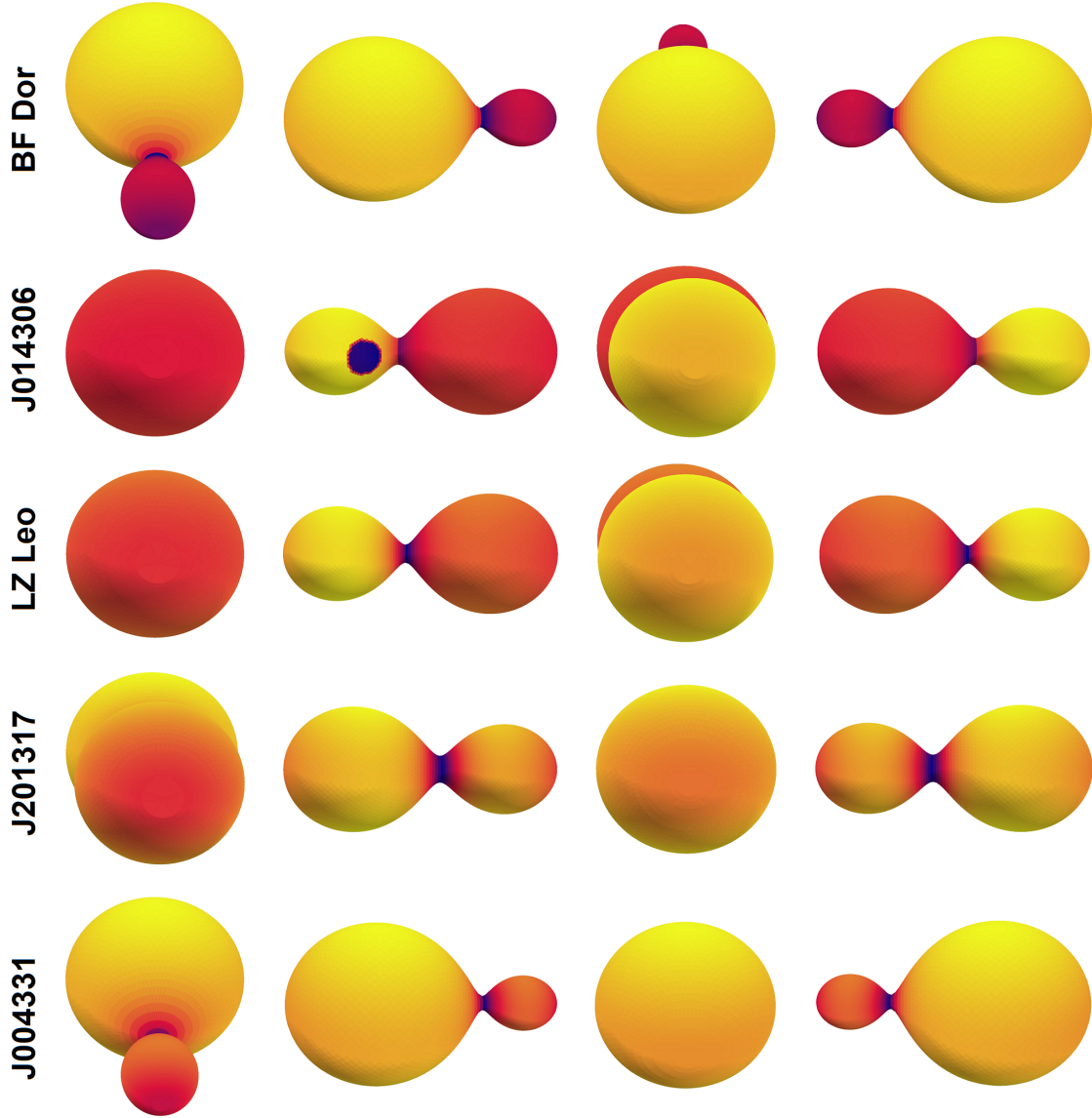


Figure 5. Three-dimensional views of the stars in the target binary systems at orbital phases 0.0, 0.25, 0.5, and 0.75, respectively.

based on the total mass, mass ratio, and orbital period of the systems, according to Equation (7) as described by Eker et al. (2006):

$$J_0 = \frac{q}{(1+q)^2} \sqrt{\frac{G^2}{2\pi}} M^5 P. \quad (7)$$

The absolute parameters obtained for the five contact binary systems are listed in Table 9.

7. Discussion and Conclusion

In this work, we present the first analysis of photometric light curves, investigate orbital period variations, and estimate

the absolute parameters of five target contact binary stars located in the northern and southern hemisphere. The outcomes of these analyses form the basis for the following discussion and conclusions:

(A) Based on the analysis of the $O - C$ diagrams, two of the targets exhibit linear variations consistent with the available minima (BF Dor and J014306), while the other two show parabolic trends (LZ Leo and J004331), both indicating a long-term decrease in their orbital periods.

To analyze the orbital period variations of LZ Leo and J004331 targets, we assumed that the observed orbital period variations are driven by conservative mass transfer. To estimate the mass transfer rates, we employed Equation (8)

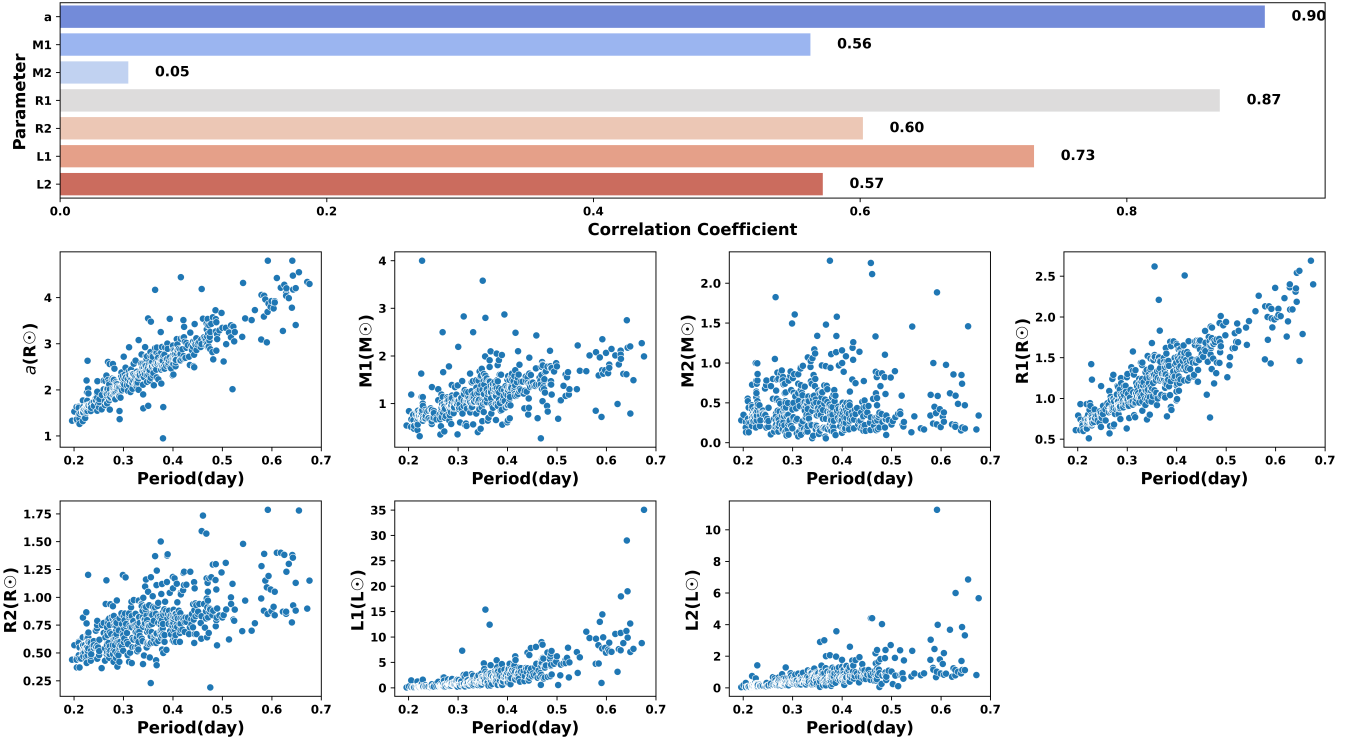


Figure 6. Correlation between orbital period and selected physical parameters of contact binaries. The top panel shows the correlation coefficients as a horizontal bar plot, while the bottom panel presents the corresponding scatter plots.

Table 9
Estimated Absolute Parameters of the Systems

Parameter	BF Dor	J014306	LZ Leo	J201317	J004331
$M_1(M_{\odot})$	1.456(415)	0.441(139)	0.550(169)	0.987(302)	1.558(425)
$M_2(M_{\odot})$	0.146(86)	0.985(383)	0.915(391)	0.480(267)	0.145(109)
$R_1(R_{\odot})$	1.444(144)	0.652(89)	0.719(79)	0.966(116)	1.593(142)
$R_2(R_{\odot})$	0.523(67)	0.935(114)	0.910(99)	0.700(95)	0.552(58)
$L_1(L_{\odot})$	1.762(535)	0.308(95)	0.337(106)	0.631(211)	1.946(540)
$L_2(L_{\odot})$	0.126(49)	0.396(109)	0.440(135)	0.310(115)	0.185(60)
$M_{\text{bol1}}(\text{mag})$	4.115(288)	6.010(291)	5.910(298)	5.230(313)	4.007(266)
$M_{\text{bol2}}(\text{mag})$	6.980(355)	5.735(265)	5.620(290)	6.001(344)	6.563(306)
$\log(g)_1(\text{cgs})$	4.282(26)	4.455(8)	4.464(26)	4.462(18)	4.226(30)
$\log(g)_2(\text{cgs})$	4.164(98)	4.490(43)	4.481(65)	4.430(81)	4.114(157)
$a(R_{\odot})$	2.455(214)	2.023(193)	2.121(198)	2.127(198)	2.695(225)
$J_0(\text{g cm}^2 \text{ s}^{-1})$	51.200(272)	51.495(216)	51.563(224)	51.538(260)	51.235(309)
$A_V(\text{mag})$	0.187(1)	0.130(1)	0.062(1)	1.323(18)	0.364(3)

(Kwee 1958):

$$\frac{\dot{P}}{P} = -3\dot{M}\left(\frac{1}{M_1} - \frac{1}{M_2}\right), \quad (8)$$

where the results of these calculations are summarized in Table 7. Given the relatively short time span covered by the

$O - C$ diagrams for all targets, additional observations will be necessary to verify these findings in the future.

(B) Light curve analysis was performed using the BSN application and the MCMC algorithm. Asymmetry in the maxima is a common feature in contact binary systems, often associated with surface phenomena such as starspots. Among the five systems studied, only one exhibited significant

Table 10
Conclusions Derived for Each Target System

Parameter	BF Dor	J014306	LZ Leo	J201317	J004331
$\Delta T = T_1 - T_2 $ (K)	780	589	258	86	310
Spectral category	G8-K3	K0-K3	K0-K2	K0-K1	G8-K1
Subtype	A	W	W	A	A
M_{1i} ($M_\odot$)	1.03	0.71	0.74	0.75	1.09
M_{2i} ($M_\odot$)	1.40	1.27	1.06	1.19	1.52
M_{lost} ($M_\odot$)	0.83	0.55	0.34	0.47	0.92

Table 11
Summary Statistics of Relative Temperature Difference in Contact Binary Systems

Statistic	Value
Median DT%	3.29%
90th percentile DT%	9.38%
95th percentile DT%	13.41%
Maximum DT%	48.83%
Systems with DT% < 1%	17.3%
Systems with DT% < 2%	32.9%
Systems with DT% < 5%	66.1%
Systems with DT% < 10%	91.1%
Systems with DT% < 15%	95.3%
Systems with DT% < 20%	96.6%

asymmetry that required the inclusion of a cold starspot in the model. This type of asymmetry is commonly referred to as the O’Connell effect, which may be explained by stellar surface inhomogeneities such as starspots (O’Connell 1951).

Based on the light curve solutions, the stellar effective temperatures of the target systems span from 4736 to 5534 K. Among them, the systems J201317 and BF Dor exhibit the smallest and largest temperature differences ($\Delta T = |T_1 - T_2|$) between their components, respectively (Table 10).

It is well established that in contact binary systems, the stellar components are expected to have close surface temperatures due to ongoing mass and energy exchange through the common envelope. Nevertheless, a statistical view of the temperature differences between the components of contact binary systems can be informative and insightful. For this purpose, we prepared a sample of 763 contact binary systems characterized by orbital periods shorter than 0.7 day and hotter components with effective temperatures below 8000 K from the Poro et al. (2025a) study. The temperature difference percentage (DT%) quantifies the relative difference in surface temperature between the two stellar components in contact binary systems. The median DT% in the sample is approximately 3.29%, indicating that half of the systems exhibit relatively small temperature contrasts. Notably, about 90% of the systems have a DT% below 9.38%, demonstrating that the vast majority of contact binaries maintain close thermal equilibrium. Furthermore, only a small fraction of

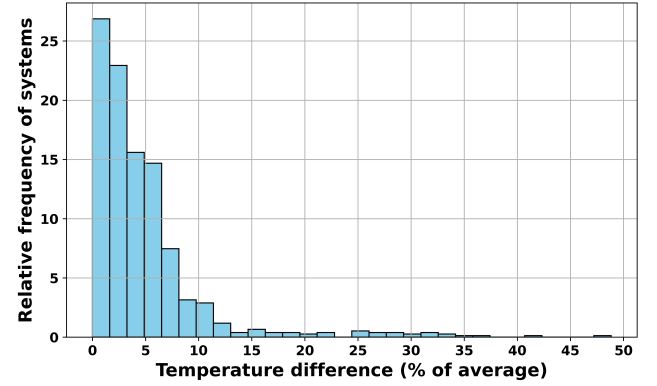


Figure 7. Histogram showing the relative frequency distribution of temperature difference percentages among contact binary systems. The x-axis represents the temperature difference as a percentage of the average component temperature, while the y-axis shows the fraction of systems in each bin.

systems show large temperature differences, with the maximum DT% reaching nearly 49%.

These results suggest that most contact binaries currently tend to have components with close or equal temperatures, although a minority exhibit significant thermal disparities. The detailed results are presented in Table 11 and Figure 7. The DT% of the five target systems—BF Dor (15.16%), J014306 (11.71%), LZ Leo (5.10%), J201317 (1.66%), and J004331 (5.91%)—span a range from close thermal equilibrium to moderately higher contrasts. Most of these systems fall within the 90th percentile of the overall sample, while BF Dor and J014306 exhibit temperature differences approaching the upper range of observed values. The relatively large effective temperature differences observed in BF Dor and J014306 may arise from different physical causes. In the case of BF Dor, the extreme mass ratio of $q = 0.1$ likely leads to inefficient energy transfer between the components, preventing them from reaching full thermal equilibrium (Fabry et al. 2023). For J014306, however, the temperature difference may instead reflect variations in the evolutionary status of the components or other system-specific factors that affect the efficiency of energy redistribution. Both systems warrant further studies to

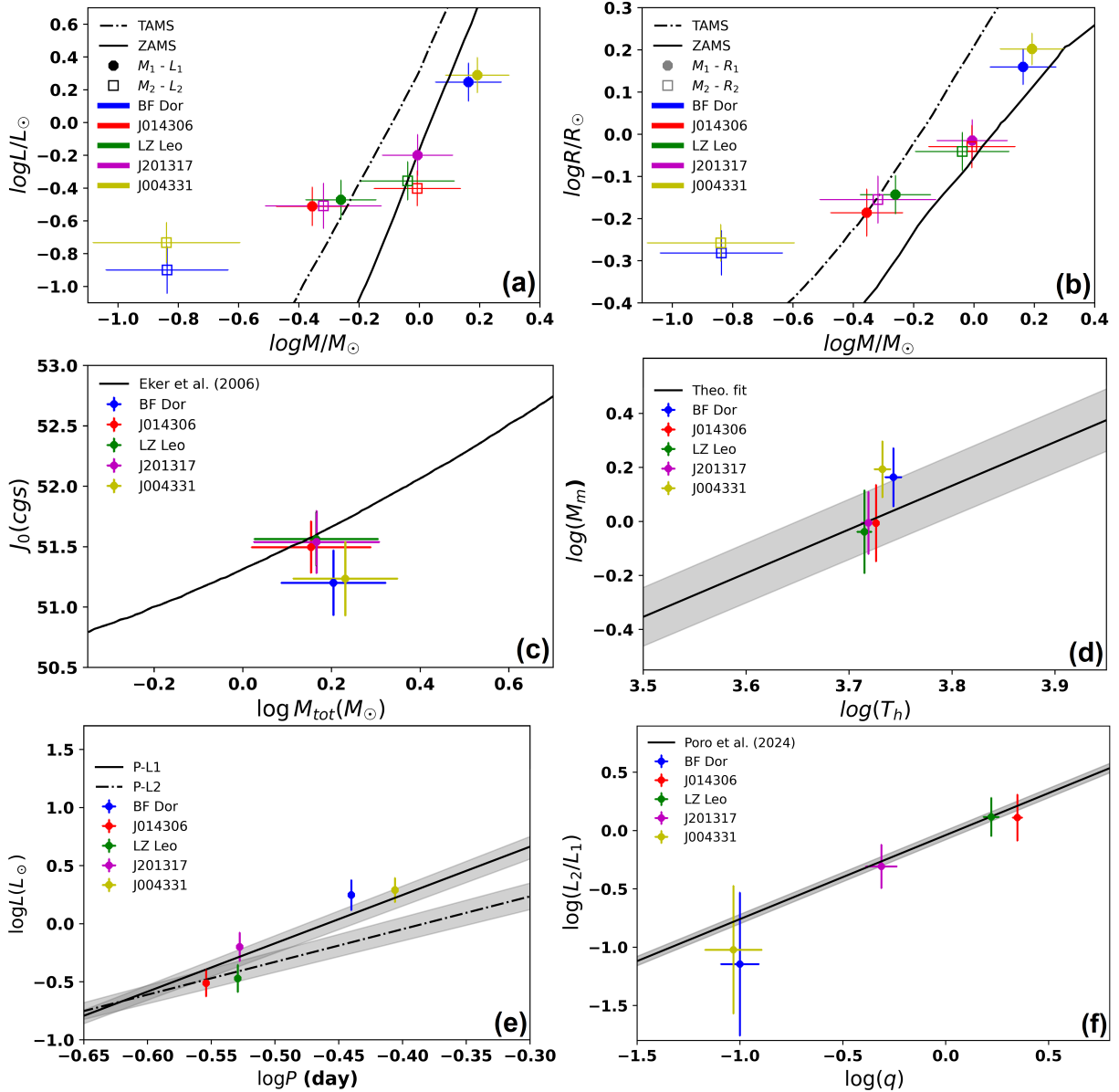


Figure 8. (a) $M-L$, (b) $M-R$, (c) $M_{\text{tot}}-J_0$, (d) T_h-M_m , (e) $P-L$, (f) $q-L_{\text{ratio}}$.

better understand the origin of these surface temperature differences.

The spectral classifications of the component stars were determined using the temperature scales provided by Cox (2000) and Eker et al. (2018), as listed in Table 10.

(C) To assess the evolutionary status of the targets, logarithmic Mass-Radius ($M-R$) and Mass-Luminosity ($M-L$) diagrams were presented based on the estimated absolute parameters (Table 9, Figure 8). The stellar components are plotted relative to the Zero-Age Main Sequence (ZAMS) and Terminal-Age Main Sequence (TAMS) as defined by Girardi et al. (2000), adopting a metallicity of $Z = 0.019$ (solar composition) along with the standard helium

abundance and mixing-length parameters used in their models. As illustrated in Figures 8(a), (b), the more massive components in all systems lie above the TAMS line, while the less massive companions are positioned around the ZAMS line. It is important to note that contact binaries result from complex binary evolution and interaction mechanisms (Yakut & Eggleton 2005; Stepień 2011), and their evolutionary paths diverge significantly from those of single stars. Consequently, any direct comparison with single-star ZAMS and TAMS lines should be approached with caution.

We used the $P-a$ empirical parameter relationship to estimate the absolute parameters of the target systems, and the system's total mass (M_{tot}) was computed. Then, the orbital

angular momentum of each system was estimated using Equation (7). We show the location of each system in the $\log M_{\text{tot}}-J_0$ diagram (Figure 8(c)), based on the results in Table 9. The area below the quadratic line in Figure 8(c) is generally associated with contact binary stars, whereas the area above corresponds to detached systems (Eker et al. 2006), although this boundary is not strictly defined.

The study by Poro et al. (2024a) used a sample of 428 contact binary systems to investigate an empirical T_h-M_m relationship, where T_h denotes the effective temperature of the hotter component and M_m refers to the mass of the more massive star. Based on the mass derived from the estimated absolute parameters and the effective temperature obtained from the light curve solution, we show the position of the star on the T_h-M_m diagram (Figure 8(d)). The component shows good agreement with the empirical relationship and its associated uncertainty.

The positions of the stars were examined with respect to two empirical parameter relationships proposed by Poro et al. (2024d): $P-L$ and $q-L_{\text{ratio}}$. As shown in Figures 8(e), (f), the stars' locations show good agreement with the empirical fits and their uncertainties.

(D) Based on the light curve solutions, which include the derived mass ratios, fillout factors, and orbital inclinations, the target systems are identified as eclipsing contact binaries. The subtype classification is determined by the effective temperature and mass of the components: if the less massive star has a higher temperature, the system is classified as W subtype; if the more massive star is hotter, it is classified as A subtype (Binnendijk 1970). According to this criterion, three of the targets are classified as A subtype, while the remaining two belong to the W subtype (Table 10).

(E) Determining the initial masses of the components in a contact binary system is essential for tracing their evolutionary paths. In this study, we applied the method proposed by Yildiz & Doğan (2013), which neglects the effects of energy transfer between the components.

To estimate the initial mass of the secondary component, we used an empirical relationship that incorporates its current mass and luminosity. The mass inferred from the luminosity, denoted as M_L , was calculated using a mass–luminosity relation (Equation (10)). The mass gained by the secondary through accretion (ΔM) was then added to its current mass (M_2) to derive the initial mass (M_{2i}) using Equation (9).

Then, the initial mass of the primary component (M_{1i}) was determined by subtracting the effective transferred mass from its current mass from Table 9, using Equation (11). This takes into account the mass lost from the system, modeled by the parameter γ , which expresses the ratio of lost mass to the total transferred mass. We adopted a value of $\gamma = 0.664$, as suggested by Yildiz & Doğan (2013). Additionally, a reciprocal mass ratio within the range $0 < 1/q < 1$ was used for consistency in the modeling.

The set of equations used in these calculations is given below:

$$M_{2i} = M_2 + \Delta M = M_2 + 2.50(M_L - M_2 - 0.07)^{0.64}, \quad (9)$$

$$M_L = \left(\frac{L_2}{1.49} \right)^{1/4.216}, \quad (10)$$

$$M_{1i} = M_1 - (\Delta M - M_{\text{lost}}) = M_1 - \Delta M(1 - \gamma). \quad (11)$$

Note that in above equations, M_1 and M_2 are the current masses of the primary and secondary components, respectively; M_{1i} and M_{2i} represent their initial masses. The term M_L corresponds to the mass estimated from the secondary's luminosity, while ΔM is the transferred mass, and M_{lost} refers to the mass expelled from the system. The resulting initial masses, listed in Table 10, show good agreement with the studies by Yildiz & Doğan (2013) and Yildiz (2014).

(F) Contact binaries with extremely low mass ratios present significant observational and theoretical challenges. The detection of reliable radial velocity curves is often difficult or even impossible due to the extremely faint and low-mass secondaries, compounded by the broad and blended spectral lines (Kjurkchieva et al. 2019). In such cases, high-quality photometric light curves become especially valuable for deriving physical parameters and analyzing the nature of these systems. These binaries are of particular interest, as they are considered likely progenitors of stellar mergers and offer key insights into the physics of binary coalescence.

Numerous theoretical studies have proposed that contact binaries possess a lower-limit cut-off in mass ratio, below which they become dynamically unstable and may ultimately merge. An early estimate by Rasio (1995), which ignored the spin angular momentum of the secondary, placed this limit at approximately $q_{\text{min}} \sim 0.09$. Subsequent models that included additional angular momentum effects have suggested lower thresholds: Li & Zhang (2006) proposed $q_{\text{min}} \sim 0.076-0.078$; Arbutina (2007) and Arbutina (2009) found $q_{\text{min}} \sim 0.070-0.109$; and Jiang et al. (2010) estimated $q_{\text{min}} \sim 0.05-0.105$, depending on the internal structure of the primary. A statistical analysis by Yang & Qian (2015) yielded $q_{\text{min}} \approx 0.044$, based on the relations between $q-f$ and $q-J_{\text{spin}}/J_{\text{orb}}$. More recently, Zhang (2024) and Li et al. (2024) proposed even lower thresholds of $q_{\text{min}} \approx 0.038-0.041$ and $q_{\text{min}} = 0.0356$, respectively.

BF Dor and J004331 target systems are classified as contact binaries with extremely low mass ratios, estimated to be 0.10 for BF Dor and 0.093 for J004331 in this study. These values place them very close to the theoretical threshold for orbital instability, below which contact binaries are expected to become dynamically unstable and eventually merge. The proximity of BF Dor and J004331 to this limit suggests that they may be in advanced evolutionary stages, making them important targets for testing predictions of binary instability and merger scenarios.

Assessing the dynamical stability of the contact binary systems BF Dor and J004331 requires evaluating the ratio of spin angular momentum (J_{spin}) to orbital angular momentum, as described by Hut (1980). In this study, we employed the equation provided by Yang & Qian (2015) to compute J_{spin}/J_0 for the two systems analyzed:

$$\frac{J_{\text{spin}}}{J_0} = \frac{1+q}{q} [(k_1 r_1)^2 + (k_2 r_2)^2 q], \quad (12)$$

where, k_1 and k_2 denote the dimensionless gyration radii, and r_1 and r_2 represent the relative radii of the components. The adopted values for k_1 and k_2 are taken from Li & Zhang (2006). We found that the values of J_{spin}/J_0 are 0.0193 for BF Dor and 0.0207 for J004331, suggesting that both systems are dynamically stable in terms of their J_{spin}/J_0 ratios (Li & Zhang 2006). According to the study by Wadhwa et al. (2021), however, our calculations imply that both systems would be dynamically unstable if their instability mass ratios were smaller than ≈ 0.075 for BF Dor and ≈ 0.070 for J004331.

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Data Availability

Ground-based data are available in the paper's online supplement.

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